TEACHING MANIPULATIVES

Curated by

THE COMPLETE TEACHER

Using manipulatives

Learn how to effectively use fraction strips, spinners, counters, and more

Manipulatives are physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics. They can be used to introduce, practice, or remediate a concept. Use this resource to help your students learn how to use manipulatives successfully

What is it?

Manipulatives are physical objects that are used as teaching tools to engage students in the hands-on learning of mathematics. They can be used to introduce, practice, or remediate a concept. A manipulative may be as simple as grains of rice or as sophisticated as a model of our solar system. They may be store-bought, brought from home, or teacher- or student-made.

They can be used in all areas of math instruction-teaching number and operations, algebra, geometry, measurement, and so on-and across all grade levels. The following are a few manipulatives that might be used at the elementary level.

Number and Operations

- Counters can be used to teach one-to-one correspondence, ordinal numbers, and basic addition and subtraction.
- Two-sided counters can be used to model one-to-one correspondence, addition and subtraction, or skip-count.
- Place-value mats can be used to show each digit's place value when using base-10 blocks.
- Base-10 blocks can be used to model the variety of ways a number can be represented, and to model regrouping when adding, subtracting, multiplying, and dividing.
- Money, in coins or note, can be used to count, skip-count, or model regrouping when adding or subtracting.

- Fraction strips can be used to show equivalent fractions, to add fractions, to subtract fractions, or to find common denominators.

Algebra

- Pattern blocks can be used to create, find, or extend patterns.
- Attribute blocks can be used to sort and classify according to shape, color, size, or other attributes.
- Pan balances or scales can be used with objects or weights to show equivalent values.

Geometry

- Geoboards can be used to identify simple geometric shapes and describe their properties and to develop spatial sense.
- Geometric-solid models can be used to teach nets or spatial reasoning.

Measurement

- Standard and non-standard rulers and measuring cups can be used to represent length or volume.
- Tiles can be used to find the area or the perimeter of an object.

Data analysis and Probability

- Spinners can be used to find the experimental probability of landing on a designated area.
- Number cubes or dice can be used to find the experimental probability of rolling a certain number or a combination of numbers.

Why is it important?

According to learning theory based on psychologist Jean Piaget's research, children are active learners who master concepts by progressing through three levels of knowledge concrete, pictorial, and abstract. The use of manipulatives enables students to explore concepts at the first, or concrete, level of understanding. When students manipulate objects, they are taking the necessary first steps toward building understanding and internalising math processes and procedures. For example, when learning to add fractions, students can use fraction strips to represent each addend. They then add the

fractional parts to find the sum. After practicing with these, they can progress to finding sums for problems on paper, represented by pictures with corresponding fraction numerals (pictorial level).

Over time, they will devise strategies and apply algorithms so they can find sums when given only the addition expression (abstract level).

For a learner to understand and handle a concept a the abstract or symbolic level successfully, he or she must first understand the concept at the other two levels, in the order given-concrete then pictorial. Using concrete manipulatives is the first step to using mental images and models. When students demonstrate understanding with the concept at this physical, or concrete, level then they are ready to move to the next level, where they can apply their knowledge using representations of the objects in place of the objects themselves.

Young children often solve addition and subtraction problems by counting concrete objects, beginning with their fingers. They go on to use concrete objects such as base-ten blocks and counters to develop more sophisticated problem-solving strategies based on what they know about counting (Siegler 1996).

Concrete models can help students represent numbers and develop number sense; they can also help bring meaning to written symbols and can be useful in building place-value concepts. You should try to "get inside students' heads" as they work with concrete materials by asking questions that elicit their thinking and reasoning. In this way, you can get a better sense of what students know and don't know, along with identifying misconceptions, thereby developing a basis for intervention strategies.

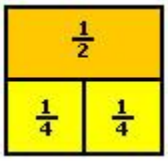
You should also choose interesting tasks using manipulatives (such as an abacus) that engage students in mathematical thinking and reasoning, which builds their understanding of numbers as well as relationships among numbers (NCTM 2000).

In fact, research shows that using manipulatives can contribute to the development of well-grounded, interconnected understandings of mathematical ideas. Students can more easily remember what they did and explain what they were thinking when they used manipulatives to solve a problem (Stein 2001).

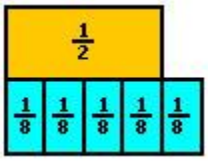
How can you make it happen?

The amount of time students need to progress from concrete to abstract understanding varies by concept and by student. It may occur within a single lesson, but more often it occurs over days, weeks, months, or even years, as the concept is revisited. A concept (for example, multiplication) may be introduced and practiced at the concrete and pictorial levels in one grade, then reviewed at these levels and practiced at the abstract level in the next grade.

Students in primary grades need to see many examples of the same concept as well as a variety of non-examples of the concept.For example, these are equivalent fractions: 1/2 = 1/4



These are not equivalent fractions: 1/2, 5/8



When students can use different manipulatives to represent the same concept, their ability to understand subsequent math concepts is enhanced. When a new manipulative is going to be used in a lesson, you should allow students time to examine it and explore its use before giving them concrete directions. You can do this by allowing time at the beginning of the lesson for this unstructured use, or by making the manipulative available to students in an area of the classroom they have access to during the day, such as in a learning center. Then, before you give students the task they are to work on using the manipulative, demonstrate how to use it.

For example, when using base-10 blocks, introduce the ones-blocks, model how to count them, and show students where they are placed on the place value chart. Then, show students the ten-sticks. Have them count each block in the ten-stick to discover that there are ten blocks, and then show students where they are placed on the place-value chart. Have student pairs regroup ten-sticks into ones-blocks and model numbers such as 11, 16, or 23.

In upper-level classes, lessons are more productive when teachers structure activities and allow some systematic exploration of the manipulatives, when teachers do not immediately correct deviations from the prescribed "procedure," and when they guide rather than lead exploration of mathematical ideas (Stein 2001).

Whole-number operations

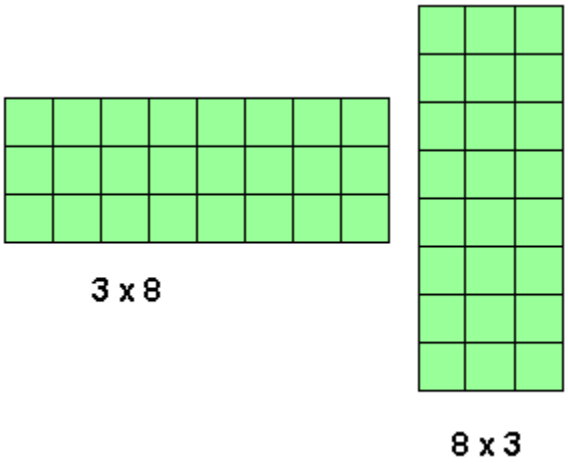
You can model conventional ways of representing mathematical situations, but it is important for students to use representations that are meaningful to them. For example, to find the sum of 43 + 29, some students prefer to add the numbers in the tens place, 40 + 20, add the numbers in the ones place, 9 + 3, and combine them, 60 + 12 = 72, to find the sum. Introduce students to manipulatives, and have students use them to solve a problem using conventional algorithms or their own, and record their work. Students in grades 3-5 should use manipulatives as tools for solving problems, such as blocks to represent place value, counters to represent multiplication, and graphs and charts to model problems.

Place value

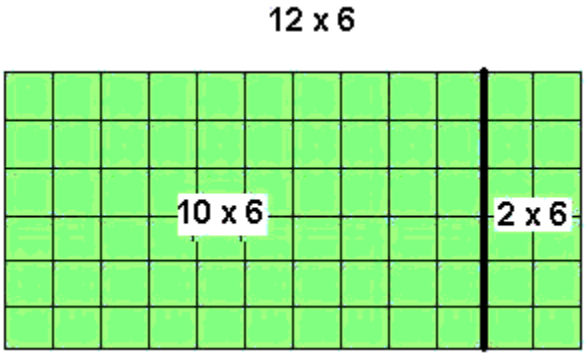
Have students use base-10 blocks to represent numbers in as many ways as possible. For example, the number 38 can be made with 3 ten-sticks and 8 one-blocks, 2 ten-sticks and 18 one-blocks, 38 one-blocks, and so on. Students can use the place value charts to write numbers using expanded notation:
38 is 3 tens and 8 ones
30 + 8.

Multiplication

Have students use counters to make arrays representing multiplication facts. Make an array of counters for 3 x 8, and have students record the multiplication sentence and product. Have them also make an array of 8 x 3 to reinforce their understanding of the commutative property when solving multiplication problems.



Arrays can also help students understand the distributive property. Students can find 12 x 6 by multiplying 10 by 6, and then 2 by 6, and then adding the products together to find the answer.



Money

Have students use coins and notes to understand the real-world use of regrouping, skip-counting, and other mathematical concepts. Have students find all the ways to show $1.00, how to regroup a dollar in order to give 4 friends 25 cents, how to make change, and how to skip-count to find the value of a group of nickels or dimes.

Geometry/Spatial reasoning

You should encourage students to explore shapes and their attributes and provide them with appropriate manipulatives and a structured environment to make this happen. Students can explore shapes by decomposing them, creating new shapes, and comparing and sorting them. Students can create shapes on geoboards, dot paper, or graph paper, and represent them in drawings or blocks.

Sort by attributes

For primary students, use different shapes in different colors and sort them by attribute. Have students determine how to classify the pieces, by color, shape, size, and so on.

Pattern Blocks/Tangrams

Have students explore how shapes can be combined to make other shapes. For example, how many triangles are in a rectangle?

Fractions and decimals

Students need to be able to build on a solid conceptual understanding of place value and whole numbers when learning about fractions and decimals. Without this background, students may not understand how 1.0 is greater than .900, or that 1/3 is more than 5/6. In grades 3-5, students learn about, compare, and find equivalent forms of fractions. Students can use manipulatives to understand how two fractions can have the same value, as well as how they relate to whole numbers. Using a variety of manipulatives can help students compare fractions to decimal values

Fraction strips

Have students compare other fraction strips to the benchmarks of 1 whole, ½ and ¼, to discover which is a greater or lesser value.

Decimal models

Have students use decimal place-value charts and base 10 blocks to model decimal numbers. They can also use a hundreds chart and shade the blocks to represent 10 of 100, 50 of 100, and so on.

Recording and communicating mathematical thinking

Manipulatives also help students to communicate their thinking processes. Discussion may be between students working in pairs, among students working in groups, or between students and the teacher as he or she circulates around the room to monitor students' work. Large-group, teacher-guided discussions should also take place at the end of the lesson. This is an opportunity for students to share their strategies and findings and summarise their experiences. It's also an opportunity for you to assess students' understanding, challenge their thinking through questioning, and highlight important aspects of the lesson.

Students can record mathematical ideas by writing the solution to a problem and the steps they took to find it. For example, students who are using place-value blocks can write number sentences to correspond to the adding being done with the manipulatives. Students should also summarise their findings such as, "The circumference of each circle I measured was always a little more than three times the length of the circle's diameter." Students should also support their conclusions. In recording their work, students are beginning the transition into the next level of understanding, pictorial.

Assessment

Students' representations often provide an insight into their mathematical thinking. Listen to how students describe their representations and the process they used, and analyse their reasoning to assess learning.

Remediation

Manipulatives can also be used in remediation. If a student is not performing well at one of the subsequent levels, pictorial or abstract, it may be helpful to go back to the concrete level to give more practice at this prerequisite level. This gives both you and the student an opportunity to correct any misconceptions that were carried into the other levels or to identify important knowledge that was not carried forward.

How can you stretch students' thinking?

Help students extend the knowledge they are building, using manipulatives, to a subsequent or lateral math topic.

For example, if students are using place value blocks to subtract, help them extend what they are learning about regrouping to subtracting with larger numbers. For example, if students are solving the problem 43 - 16, they know they can regroup one of the four ten-sticks to 10 ones, so 43 is represented by 3 ten-sticks and 13 ones-blocks. Discuss with students how they might use that information to find the answer to the problem 430 - 160.

If students are using tiles to build various rectangles to explore area and perimeter, extend their thinking to square numbers by challenging them to find which numbers of tiles can be used to build squares. For example, the area of the shape on the right is 6 tiles, and the area of the shape on the left is 9 tiles. Have students create shapes and determine what is the same about all of the square shapes they create.

When can you use it?

Manipulatives can be used to teach basic number operations, geometry, probability, measurement, or algebra. Students can draw a picture of how they used manipulatives to solve a problem, or create their own word problems and model them. Older students can write about how they solved a math problem. If they are using manipulatives to solve 4.5 + 235.6, students can write about the step-by-step process they used to solve

the problem. Using manipulatives is also a science strategy: when students grow seeds, use wires and batteries to light bulbs, or mix solutions to determine whether liquids are acids or bases, they are involved at the hands on, or concrete, level of building understanding. The activities and experiments can help students lay the foundation of understanding scientific concepts and processes.

The following are some examples how to use manipulatives:

- Have students show three blue counters, and then match each blue counter with a red counter.
- Create a sentence such as, "____students were playing kickball and ____ students were on the swings. There were _____ students all together." Have students take turns changing the numbers, and model each sentence using manipulatives.
- Show five red counters and four blue counters. Then ask, "Are there more red or blue counters?"
- Have students use 2-sided counters to show 3 (white) + 6 (black). Then ask, "How many more black counters are there?
- Have students sort money into pennies, nickels, dimes, and quarters. How many pennies are heads up? How many nickels?
- Have students show these math problems using manipulatives. Jack collected 66 baseball cards this month. If he collects the same amount each month, how many will he have in 6 months? 66 x 6 = 396
- Sue has 10 markers. She has 2 friends. Show how many markers each friend will get.
- Sue has 10 markers. She wants each friend to get 2 markers. Show how many friends can get markers.
- There are 245 students in the second grade. If they are divided into two groups to watch a play, how many students will be in each group? 245 divided by 2 = 122 R1. There will be 122 students in one group and 123 students in the other group.

- Four t-shirts are on sale for $6.00. How much does each one cost? 600 divided by 4 = 150. Each shirt is $1.50.
- Find fractions that are equivalent to 1. Equivalent fractions: 4/4, 2/2, 8/8
- Add 1/4 + 3/6. Find the common denominator and then add. The common denominator for 4 and 6 is 12. Change 1/4 to equivalent fraction 3/12. Change 3/6 to equivalent fraction 6/12. The answer is 9/12. Change to equivalent fraction 3/4. Alternate: Change 3/6 to equivalent fraction 1/2. The common denominator for 4 and 2 is 4. Change 1/2 to 2/4, and add to find 3/4.

Lesson plans

- Comparative Subtraction - In this primary lesson, students use two-sided counters to solve subtraction problems.
- Odd and even numbers - In this primary lesson, students use two-sided counters and a hundreds chart to show and then extend the alternating pattern of odd and even numbers.
- More than, less than - This primary lesson uses base-10 manipulatives to identify the number that is one more or one less than a given two-digit number.

Source:

Created by: The Complete Teacher 2022