Everything Maths for Grade 10

Teacher's Guide

An Introduction to using Everything Maths for Grade 10

Written by Volunteers

ISBN: 978-1-920423-09-4

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

SHUTTLEWORTH
FUNDED

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More than a regular school textbook

Everything Maths is not just a Mathematics textbook. Even though it has everything you expect from a printed school textbook, it comes with a whole lot more! For a start, your learners can download it or read it on-line on their mobile phone, computer or iPad, which means everyone has the convenience of accessing it wherever they are.

It is good for learners to hear and read different explanations of concepts as it affords them a more well-rounded understanding of the work. This is why every chapter comes with links to on-line video lessons and explanations, which help bring the ideas and concepts covered to life. Summary presentations at the end of every chapter offer an overview of the content covered, with key points highlighted for easy revision.

All the exercises inside the book link to an on-line service where learners can get more practice, see the full solutions or test their skills level on mobile and PC. Every educator knows that the key to success in physical science is practice, practice, practice!

We are interested to know what you as an educator think about our books, as well as what the learners wonder about or struggle with as they make their way through the content and attempt the exercises. That is why we have made it possible for educators and learners to use their mobile phones or computers to access the books on-line and digitally pin a question to a page and see what questions and answers other readers pinned up too.

Read it on your mobile or PC

Learners can have this textbook at hand wherever they are – whether at home, on the the train or at school. They can browse the on-line version of *Everything Maths* on their mobile phone, tablet or computer. To read it off-line, a PDF or e-book version can be downloaded. Learners can access *Everything Maths* and *Everything Science* for Grade 10, 11 and 12 on their mobile phone! There is now no excuse for a learner to not have these textbooks close at hand!

To read or download the textbook on their phone or computer, direct learners to www.everythingmaths.co.za

Links to support materials

Inside the textbook you will find these icons to help you and your learners spot where on-line videos, presentations, practice tools and additional help exists. The short-codes next to the icons allow both educators and learners to navigate directly to the resources on-line without having to spend time searching for them. Visit www.everythingmaths.co.za and enter the short-codes in the navigation box.

www **(A123)** – Go directly to a section

(V123) – Video, simulation or presentation

(P123) – Practice and test your skills

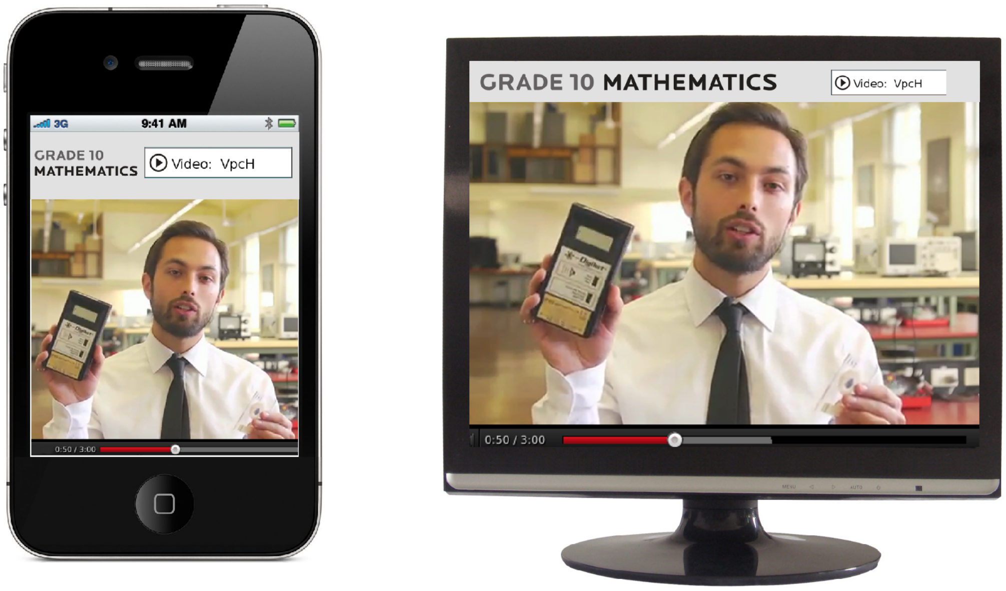
(Q123) – Ask for help or find an answer

Video lessons

Look out for the video icons inside the book. These will take you to on-line video lessons that help bring the ideas and concepts on the page to life. Learners can now get extra insight, detailed explanations and worked examples, while also seeing the concepts in action and hearing real people talk about how they use maths and science in their work! Included in this are the interactive PhET Simulations, which offer an interactive and accurate representation of physics and chemistry concepts, allowing learners to manipulate the variables and see immediate results. These are good fun and a great way to get your learners hands on with physical science!

This is a great way for you to bring technology into your classroom – using a projector or digital whiteboard, access the books on www.everythingmaths.co.za and use the videos to provide an additional summary of the concepts you have covered by offering an alternative explanation. After hours, learners that need additional help will know that they can watch the videos in their own time, with the added bonus of being able to stop, pause and rewind the explanation until they have fully grasped the concept. This is great for revision purposes too, as it is like having a personal teacher on hand for every learner, at any time!

See video explanation (Video : V123)



You can access these videos by:

- viewing them on-line on your phone or computer
- downloading the videos for off-line viewing on your phone or computer
- ordering a DVD to play on your TV or computer
- downloading them off-line over Bluetooth or Wi-Fi from select outlets

For additional viewing, downloads or more information, visit the *Everything Maths* website on your phone or computer at www.everythingmaths.co.za

Video exercises

Wherever there are exercises in the book you will see icons and short-codes for on-line video solutions, practice and help. By entering these short-codes into the box on our website, learners will be taken to video solutions of select exercises to show them step-by-step how to solve such problems. Encourage your learners to access these video exercises, which are great for revision purposes as well as to reinforce your own teaching.

See video exercise (Video : V123)

You can access these videos by:

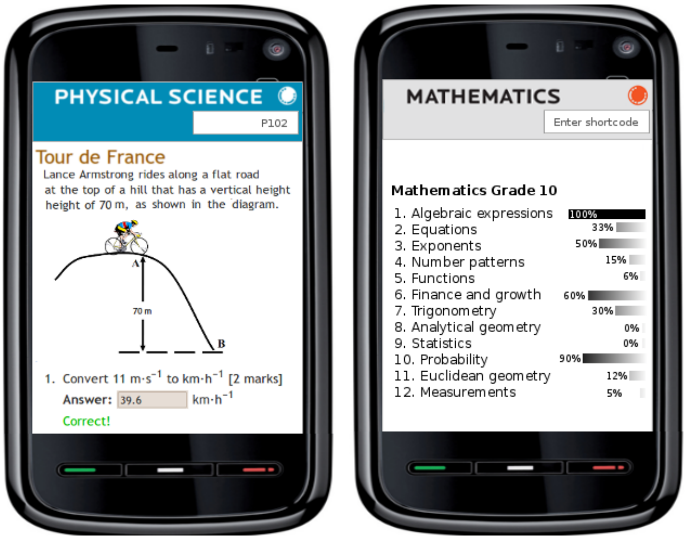
- viewing them on-line on your phone or computer
- downloading the videos for off-line viewing on your phone or computer
- ordering a DVD to play on your TV or computer
- downloading them off-line over Bluetooth or Wi-Fi from select outlets

For additional viewing, downloads or more information, visit the *Everything Maths* website on your phone or computer at www.everythingmaths.co.za

Intelligent Practice for learners

One of the best ways for learners to prepare for tests and exams is to practice answering the same kind of questions they will be tested on. At every set of exercises in the textbook you will see a practice icon and short-code, which link to an on-line database for learners to practice further exercises. Point your learners at www.everythingmaths.co.za on their mobile phone or PC, where they can enter the short-code from the textbook into the box on the website, and be redirected to additional exercises on-line. This on-line practice on **mobile** and **PC** will keep track of learners' performance and progress, give them feedback on areas which require more attention, and suggest which sections or videos to look at.

See more practice (QM123)



The software can generate any number of questions with the same structure but different details i.e. the numerical values in physics or maths problems can change each time, but the type of question can stay the same. This allows much more variety than a traditional question bank, to the extent that a different practice test can be created automatically for each student in a class. The system also generates a memorandum along with each test, and tracks the learners' conceptual understanding through their success at answering different types of questions.

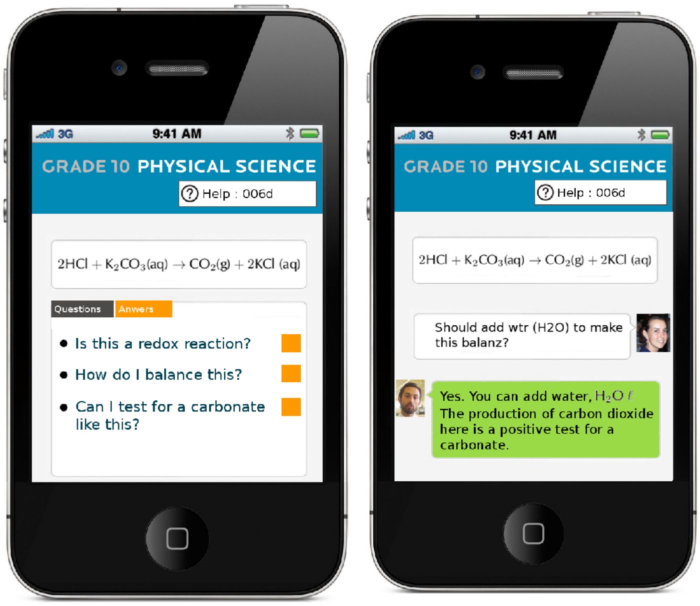
This tool aims to discover the strong and weak points in learners' understanding as the learners are going through worked examples and drilling exam problems. By knowing with which concepts learners are struggling, the system can then do useful things like

- provide more practice on the types of questions with which the learner is struggling;
- recommend revision material from freely available educational resources (for example, Siyavula's *Everything Maths* and *Everything Science* textbooks);
- provide feedback and reports to learners, educators and parents about their progress and about the specific concepts to which they should pay more attention.

The above is done for each learner individually, delivering a customised practice and revision schedule to match his or her pace and understanding.

Helping learners find answers

When a learner is stuck on a particular section of work in the textbook, they can get additional help by visiting www.everythingmaths.co.za on their phone or computer, and find out if other learners also had a question about that same section of the work. If a question has been posted, educators can go on-line and respond, thereby helping other learners that may have been stuck on the same problem.



Database of questions and answers

We invite learners to browse our database of questions and answer for every sections and exercises in the book. They can use the short-code for the section or exercise where they have a question and enter it into the short-code search box on the web or mobi-site at www.everythingmaths.co.za or www.everythingscience.co.za. They will be directed to all the questions previously asked by learners and answered by experts for that section or exercise.

www **(P78) Visit this section to post or view questions**

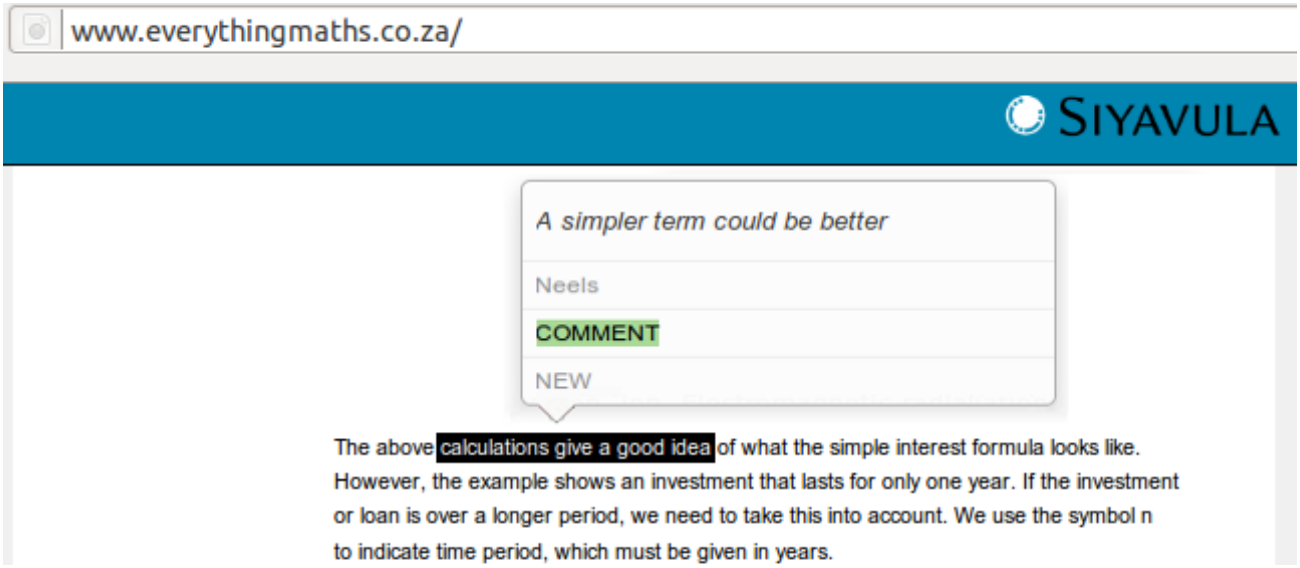
Ask an expert

They can't find their question or the answer to it in the questions database? Then we invite them to try our service where they can send their question directly to an expert who will reply with an answer. Again,they use the short-code for the section or exercise in the book to identify communicate their problem area.

(QM123) Help or ask a question

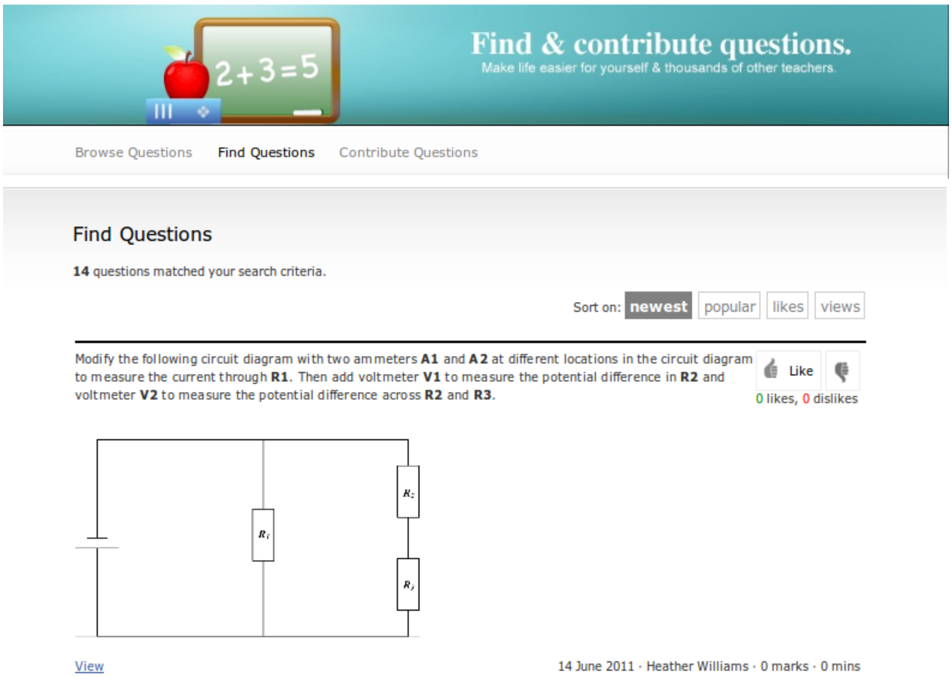
Tell us how to improve the book

If you have any comments, thoughts or suggestions on the books, visit www.everythingmaths.co.za , and switch to educator mode (you will see on the website how to do this), and using our annotator tool, you can capture these in the text. These can range from sharing tips and ideas on the content in the textbook with your fellow educators, to discussing how to better explain concepts in class. Also, if you have picked up any errors in the book you can make a note of them here, and we will correct them in time for the next print run. The image below illustrates this.



Setting tests with Monassis

Siyavula offers an open online assessment bank called Monassis, for the sharing and accessing of curriculum-aligned test and exam questions with answers. All the questions and solutions found in the textbook are hosted on-line on Monassis. In addition to this, this site enables educators to quickly set tests and exam papers, by selecting items from the library and adding them to their test. Educators can then download their separate test and memo which is ready for printing. Monassis further offers educators the option of capturing their learners' marks in order to view a selection of diagnostic reports on their performance.



We encourage you to make use of Monassis - let it help you save time setting tests and analysing learner marks! Visit Monassis at http://www.monassis.com

Part I

Introduction

Front matter 1

Support for educators

Science education is about more than physics, chemistry and mathematics... It's about learning to think and to solve problems, which are valuable skills that can be applied through all spheres of life. Teaching these skills to our next generation is crucial in the current global environment where methodologies, technology and tools are rapidly evolving. Education should benefit from these fast moving developments. In our simplified model there are three layers to how technology can significantly influence your teaching and teaching environment.

First Layer: educator collaboration

There are many tools that help educators collaborate more effectively. We know that communities of practice are powerful tools for the refinement of methodology, content and knowledge and are also superb for providing support to educators. One of the challenges facing community formation is the time and space to have sufficient meetings to build real communities and exchange practices, content and learnings effectively. Technology allows us to streamline this very effectively by transcending space and time. It is now possible to collaborate over large distances (transcending space) and when it is most appropriate for each individual (transcending time) by working virtually (email, mobile, online etc.).

Our textbooks have been re-purposed from content available on the Connexions website (http://cnx.org/lenses/fhsst). The content on this website is easily accessible and adaptable as it is under an open licence, stored in an open format, based on an open standard, on an open-source platform, for free, where everyone can produce their own books. The content on Connexions is available under an open copyright license - CC-BY. This Creative Commons By Attribution Licence allows others to legally distribute, remix, tweak, and build upon the work available, even commercially, as long as the original author is credited for the original creation. This means that learners and educators are able to download, copy, share and distribute this content legally at no cost. It also gives educators the freedom to edit, adapt, translate and contextualise it, to better suit their teaching needs.

Connexions is a tool where individuals can share, but more importantly communities can form around the collaborative, online development of resources. Your community of educators can therefore:

- form an online workgroup around the content;
- make your own copy of the content;
- edit sections of your own copy;
- add your own content or replace existing content with your own content;
- use other content that has been shared on the platform in your own work;
- create your own notes / textbook / course material as a community.

Educators often want to share assessment items as this helps reduce workload, increase variety and improve quality. Currently all the solutions to the exercises contained in the textbooks have been uploaded onto our free and open online assessment bank called Monassis (www.monassis.com), with each exercise having a shortcode link to its solution on Monassis. To access the solution simply go to www.everythingmaths.co.za, enter the shortcode, and you will be redirected to the solution on Monassis.

Monassis is similar to Connexions but is focused on the sharing of assessment items. Monassis contains a selection of test and exam questions with solutions, openly shared by educators. Educators can further search and browse the database by subject and grade and add relevant items to a test. The website automatically generates a test or exam paper with the corresponding memorandum for download.

By uploading all the end-of-chapter exercises and solutions to this open assessment bank, the larger community of educators in South Africa are provided with a wide selection of items to use in setting their tests and exams. More details about the use of Monassis as a collaboration tool are included in the Monassis section.

Second Layer: classroom engagement

In spite of the impressive array of rich media open educational resources available freely online (such as videos, simulations, exercises and presentations), only a small number of educators actively make use of them. Our investigations revealed that the overwhelming quantity, the predominant international context, and difficulty in correctly aligning them with the local curriculum level acts as deterrents. The opportunity here is that, if used correctly, they can make the classroom environment more engaging.

Presentations can be a first step to bringing material to life in ways that are more compelling than are possible with just a blackboard and chalk. There are opportunities to:

- create more graphical representations of the content;
- control timing of presented content more effectively;
- allow learners to relive the lesson later if constructed well;
- supplement the slides with notes for later use;
- embed key assessment items in advance to promote discussion; and
- embed other rich media like videos.

Videos have been shown to be potentially both engaging and effective. They provide opportunities to:

- present an alternative explanation;
- challenge misconceptions without challenging an individual in the class; and
- show an environment or experiment that cannot be replicated in the class which could be far away, too expensive or too dangerous.

Simulations are also very useful and can allow learners to:

- have increased freedom to explore, rather than reproduce a fixed experiment or process;
- explore expensive or dangerous environments more effectively; and
- overcome implicit misconceptions.

We realised the opportunity for embedding a selection of rich media resources such as presentations, simulations, videos and links into the online version of Everything Maths and Everything Science at the relevant sections. This will not only present them with a selection of locally relevant and curriculum aligned resources, but also position these resources within the appropriate grade and section. Links to these online resources are recorded in the print or PDF versions of the books, making them a tour-guide or credible pointer to the world of online rich media available.

Third Layer: beyond the classroom

The internet has provided many opportunities for self-learning and participation which were never before possible. There are huge stand-alone archives of videos like the Khan Academy which covers most Mathematics for Grades 1 - 12 and Science topics required in FET. These videos, if not used in class, provide opportunities for the learners to:

- look up content themselves;
- get ahead of class;
- independently revise and consolidate their foundation; and
- explore a subject to see if they find it interesting.

There are also many opportunities for learners to participate in science projects online as real participants (see the section on citizen cyberscience "On the web, everyone can be a scientist"). Not just simulations or tutorials but real science so that:

- learners gain an appreciation of how science is changing;
- safely and easily explore subjects that they would never have encountered before university;
- contribute to real science (real international cutting edge science programmes);
- have the possibility of making real discoveries even from their school computer laboratory; and
- find active role models in the world of science.

In our book we've embedded opportunities to help educators and learners take advantage of all these resources, without becoming overwhelmed at all the content that is available online.

Tell us how to improve the book

If you have any comments, thoughts or suggestions on the books, you can visit www.everythingmaths.co.za and in educator mode (you will see on the website how to do this), capture these in the text. These can range from sharing tips and ideas on the content in the textbook with your fellow educators, to discussing how to better explain concepts in class. Also, if you have picked up any errors in the book you can make a note of them here, and we will correct them in time for the next print run.

On the Web, everyone can be a scientist

Did you know that you can fold protein molecules, hunt for new planets around distant suns or simulate how malaria spreads in Africa, all from an ordinary PC or laptop connected to the Internet? And you don't need to be a certified scientist to do this. In fact some of the most talented contributors are teenagers. The reason this is possible is that scientists are learning how to turn simple scientific tasks into competitive online games.

This is the story of how a simple idea of sharing scientific challenges on the Web turned into a global trend, called citizen cyberscience. And how you can be a scientist on the Web, too.

Looking for Little Green Men

A long time ago, in 1999, when the World Wide Web was barely ten years old and no one had heard of Google, Facebook or Twitter, a researcher at the University of California at Berkeley, David Anderson, launched an online project called SETI@home. SETI stands for Search for Extraterrestrial Intelligence. Looking for life in outer space.

Although this sounds like science fiction, it is a real and quite reasonable scientific project. The idea is simple enough. If there are aliens out there on other planets, and they are as smart or even smarter than us, then they almost certainly have invented the radio already. So if we listen very carefully for radio signals from outer space, we may pick up the faint signals of intelligent life.

Exactly what radio broadcasts aliens would produce is a matter of some debate. But the idea is that if they do, it would sound quite different from the normal hiss of background radio noise produced by stars and galaxies. So if you search long enough and hard enough, maybe you'll find a sign of life.

It was clear to David and his colleagues that the search was going to require a lot of computers. More than scientists could afford. So he wrote a simple computer program which broke the problem down into smaller parts, sending bits of radio data collected by a giant radio-telescope to volunteers around the world. The volunteers agreed to download a programme onto their home computers that would sift through the bit of data they received, looking for signals of life, and send back a short summary of the result to a central server in California.

The biggest surprise of this project was not that they discovered a message from outer space. In fact, after over a decade of searching, no sign of extraterrestrial life has been found, although there are still vast regions of space that have not been looked at. The biggest surprise was the number of people willing to help such an endeavour. Over a million people have downloaded the software, making the total computing power of SETI@home rival that of even the biggest supercomputers in the world. David was deeply impressed by the enthusiasm of people to help this project. And he realized that searching for aliens was probably not the only task that people would be willing to help with by using the spare time on their computers. So he set about building a software platform that would allow many other scientists to set up similar projects. You can read more about this platform, called BOINC, and the many different kinds of volunteer computing projects it supports today, at http://boinc.berkeley.edu/.

There's something for everyone, from searching for new prime numbers (PrimeGrid) to simulating the future of the Earth's climate (ClimatePrediction.net). One of the projects, MalariaControl.net, involved researchers from the University of Cape Town as well as from universities in Mali and Senegal.

The other neat feature of BOINC is that it lets people who share a common interest in a scientific topic share their passion, and learn from each other. BOINC even supports teams – groups of people who put their computer power together, in a virtual way on the Web, to get a higher score than their rivals. So BOINC is a bit like Facebook and World of Warcraft combined – part social network, part online multiplayer game.

Here's a thought: spend some time searching around BOINC for a project you'd like to participate in, or tell your class about.

You are a computer, too

Before computers were machines, they were people. Vast rooms full of hundreds of government employees used to calculate the sort of mathematical tables that a laptop can produce nowadays in a fraction of a second. They used to do those calculations laboriously, by hand. And because it was easy to make mistakes, a lot of the effort was involved in double-checking the work done by others.

Well, that was a long time ago. Since electronic computers emerged over 50 years ago, there has been no need to assemble large groups of humans to do boring, repetitive mathematical tasks. Silicon chips can solve those problems today far faster and more accurately. But there are still some mathematical problems where the human brain excels.

Volunteer computing is a good name for what BOINC does: it enables volunteers to contribute computing power of their PCs and laptops. But in recent years, a new trend has emerged in citizen cyberscience that is best described as volunteer thinking. Here the computers are replaced by brains, connected via the Web through an interface called eyes. Because for some complex problems – especially those that involve recognizing complex patterns or three-dimensional objects – the human brain is still a lot quicker and more accurate than a computer.

Volunteer thinking projects come in many shapes and sizes. For example, you can help to classify millions of images of distant galaxies (GalaxyZoo), or digitize hand-written information associated with museum archive data of various plant species (Herbaria@home). This is laborious work, which if left to experts would take years or decades to complete. But thanks to the Web, it's possible to distribute images so that hundreds of thousands of people can contribute to the search.

Not only is there strength in numbers, there is accuracy, too. Because by using a technique called validation – which does the same sort of double-checking that used to be done by humans making mathematical tables – it is possible to practically eliminate the effects of human error. This is true even though each volunteer may make quite a few mistakes. So projects like Planet Hunters have already helped astronomers pinpoint new planets circling distant stars. The game FoldIt invites people to compete in folding protein molecules via a simple mouse-driven interface. By finding the most likely way a protein will fold, volunteers can help understand illnesses like Alzheimer's disease, that depend on how proteins fold.

Volunteer thinking is exciting. But perhaps even more ambitious is the emerging idea of volunteer sensing: using your laptop or even your mobile phone to collect data – sounds, images, text you type in – from any point on the planet, helping scientists to create global networks of sensors that can pick up the first signs of an outbreak of a new disease (EpiCollect), or the initial tremors associated with an earthquake (QuakeCatcher.net), or the noise levels around a new airport (NoiseTube).

There are about a billion PCs and laptops on the planet, but already 5 billion mobile phones. The rapid advance of computing technology, where the power of a ten-year old PC can easily be packed into a smart phone today, means that citizen cyberscience has a bright future in mobile phones. And this means that more and more of the world's population can be part of citizen cyberscience projects. Today there are probably a few million participants in a few hundred citizen cyberscience initiatives. But there will soon be seven billion brains on the planet. That is a lot of potential citizen cyberscientists. You can explore much more about citizen cyberscience on the Web. There's a great list of all sorts of projects, with brief summaries of their objectives, at http://distributedcomputing.info/ . BBC Radio 4 produced a short series on citizen science http://www.bbc.co.uk/radio4/science/citizenscience.shtml and you can subscribe to a newsletter about the latest trends in this field at http://scienceforcitizens.net/ . The Citizen Cyberscience Centre, www.citizencyberscience.net which is sponsored by the South African Shuttleworth Foundation, is promoting citizen cyberscience in Africa and other developing regions.

Blog posts

General blogs

- Teachers Monthly - Education News and Resources
  - “We eat, breathe and live education! “
  - “Perhaps the most remarkable yet overlooked aspect of the South African teaching community is its enthusiastic, passionate spirit. Every day, thousands of talented, hard-working teachers gain new insight from their work and come up with brilliant, inventive and exciting ideas. Teacher’s Monthly aims to bring teachers closer and help them share knowledge and resources.
  - Our aim is twofold...
    - To keep South African teachers updated and informed.
    - To give teachers the opportunity to express their views and cultivate their interests.”
  - http://www.teachersmonthly.com
- Head Thoughts – Personal Reflections of a School Headmaster
  - blog by Arthur Preston
  - “Arthur is currently the headmaster of a growing independent school in Worcester, in the Western Cape province of South Africa. His approach to primary education is progressive and is leading the school through an era of new development and change.”
  - http://headthoughts.co.za/

Maths blogs

- CEO: Circumspect Education Officer - Educating The Future
  - blog by Robyn Clark
  - “Mathematics teacher and inspirer.”
  - http://clarkformaths.tumblr.com/
- dy/dan - Be less helpful
  - blog by Dan Meyer
  - “I’m Dan Meyer. I taught high school math between 2004 and 2010 and I am currently studying at Stanford University on a doctoral fellowship. My specific interests include curriculum design (answering the question, "how we design the ideal learning experience

for students?") and teacher education (answering the questions, "how do teachers learn?" and "how do we retain more teachers?" and "how do we teach teachers to teach?").”
  · http://blog.mrmeyer.com

- Without Geometry, Life is Pointless - Musings on Math, Education, Teaching, and Research
  · blog by Avery
  · “I’ve been teaching some permutation (or is that combination?) of math and science to third through twelfth graders in private and public schools for 11 years. I’m also pursuing my EdD in education and will be both teaching and conducting research in my classroom this year.”
  · http://mathteacherorstudent.blogspot.com/
- Overthinking my teaching - The Mathematics I Encounter in Classrooms
  · blog by Christopher Danielson
  · “I think a lot about my math teaching. Perhaps too much. This is my outlet. I hope you find it interesting and that you’ll let me know how it’s going.”
  · http://christopherdanielson.wordpress.com
- A Recursive Process - Math Teacher Seeking Patterns
  · blog by Dan
  · “I am a High School math teacher in upstate NY. I currently teach Geometry, Computer Programming (Alice and Java), and two half year courses: Applied and Consumer Math. This year brings a new 21st century classroom (still not entirely sure what that entails) and a change over to standards based grades (sbg).”
  · http://dandersod.wordpress.com
- Think Thank Thunk – Dealing with the Fear of Being a Boring Teacher
  · blog by Shawn Cornally
  · “I am Mr. Cornally. I desperately want to be a good teacher. I teach Physics, Calculus, Programming, Geology, and Bioethics. Warning: I have problem with using colons. I proof read, albeit poorly.”
  · http://101studiostreet.com/wordpress/

Overview

Curriculum overview

Before 1994 there existed a number of education departments and subsequent curriculum according to the segregation that was so evident during the apartheid years. As a result, the curriculum itself became one of the political icons of freedom or suppression. Since then the government and political leaders have sought to try and develop one curriculum that is aligned with our national agenda of democratic freedom and equality for all, in fore-grounding the knowledge, skills and values our country believes our learners need to acquire and apply, in order to participate meaningfully in society as citizens of a free country. The National Curriculum Statement (NCS) of Grades R – 12 (DBE, 2012) therefore serves the purposes of:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment, and meaningful participation in society as citizens of a free country;
- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace; and
- providing employers with a sufficient profile of a learner's competencies.

Although elevated to the status of political icon, the curriculum remains a tool that requires the skill of an educator in interpreting and operationalising this tool within the classroom. The curriculum itself cannot accomplish the purposes outlined above without the community of curriculum specialists, material developers, educators and assessors contributing to and supporting the process, of the intended curriculum becoming the implemented curriculum. A curriculum can succeed or fail, depending on its implementation, despite its intended principles or potential on paper. It is therefore important that stakeholders of the curriculum are familiar with and aligned to the following principles that the NCS is based on:

Principle	**Implementation**
Social Transformation	Redressing imbalances of the past. Providing equal opportunities for all.
Active and Critical Learning	Encouraging an active and critical approach to learning. Avoiding excessive rote and uncritical learning of given truths.
High Knowledge and Skills	Learners achieve minimum standards of knowledge and skills specified for each grade in each subject.
Progression	Content and context shows progression from simple to complex.
Social and Environmental Justice and Human Rights	These practices as defined in the Constitution are infused into the teaching and learning of each of the subjects.
Valuing Indigenous Knowledge Systems	Acknowledging the rich history and heritage of this country.
Credibility, Quality and Efficiency	Providing an education that is globally comparable in quality.

This guide is intended to add value and insight to the existing National Curriculum for Grade 10 Mathematics, in line with its purposes and principles. It is hoped that this will assist you as the educator in optimising the implementation of the intended curriculum.

Curriculum requirements and objectives

The main objectives of the curriculum relate to the learners that emerge from our educational system. While educators are the most important stakeholders in the implementation of the intended curriculum, the quality of learner coming through this curriculum will be evidence of the actual attained curriculum from what was intended and then implemented.

These purposes and principles aim to produce learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;
- work effectively as individuals and with others as members of a team;
- organise and manage themselves and their activities responsibly and effectively;
- collect, analyse, organise and critically evaluate information;
- communicate effectively using visual, symbolic and/or language skills in various modes;
- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and
- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

The above points can be summarised as an independent learner who can think critically and analytically, while also being able to work effectively with members of a team and identify and solve problems through effective decision making. This is also the outcome of what educational research terms the "reformed" approach rather than the "traditional" approach many educators are more accustomed to. Traditional practices have their role and cannot be totally abandoned in favour of only reform practices. However, in order to produce more independent and mathematical thinkers, the reform ideology needs to be more embraced by educators within their instructional behaviour. Here is a table that can guide you to identify your dominant instructional practice and try to assist you in adjusting it (if necessary) to be more balanced and in line with the reform approach being suggested by the NCS.

	Traditional Versus Reform Practices
Values	**Traditional**– values content, correctness of learners' responses and mathematical validity of methods. **Reform** – values finding patterns, making connections, communicating mathematically and problem-solving.
Teaching Methods	**Traditional** – expository, transmission, lots of drill and practice, step by step mastery of algorithms. **Reform** – hands-on guided discovery methods, exploration, modelling. High level reasoning processes are central.
Grouping Learners	**Traditional** – dominantly same grouping approaches. **Reform** – dominantly mixed grouping and abilities.

The subject of mathematics, by the nature of the discipline, provides ample opportunities to meet the reformed objectives. In doing so, the definition of mathematics needs to be understood and embraced by educators involved in the teaching and the learning of the subject. In research it has been well documented that, as educators, our conceptions of what mathematics is, has an influence on our approach to the teaching and learning of the subject.

Three possible views of mathematics can be presented. The instrumentalist view of mathematics assumes the stance that mathematics is an accumulation of facts, rules and skills that need to be used as a means to an end, without there necessarily being any relation between these components. The Platonist view of mathematics sees the subject as a static but unified body of certain knowledge, in which mathematics is discovered rather than created. The problem solving view of mathematics is a dynamic, continually expanding and evolving field of human creation and invention that is in itself a cultural product. Thus mathematics is viewed as a process of enquiry, not a finished product. The results remain constantly open to revision. It is suggested that a hierarchical order exists within these three views, placing the instrumentalist view at the lowest level and the problem solving view at the highest.

According to the NCS:

Mathematics is the study of quantity, structure, space and change. Mathematicians seek out patterns, formulate new conjectures, and establish axiomatic systems by rigorous deduction from appropriately chosen axioms and definitions. Mathematics is a distinctly human activity practised by all cultures, for thousands of years. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.

This corresponds well to the problem solving view of mathematics and may challenge some of our instrumentalist or Platonistic views of mathematics as a static body of knowledge of accumulated facts, rules and skills to be learnt and applied. The NCS is trying to discourage such an approach and encourage mathematics educators to dynamically and creatively involve their learners as mathematicians engaged in a process of study, understanding, reasoning, problem solving and communicating mathematically.

Below is a check list that can guide you in actively designing your lessons in an attempt to embrace the definition of mathematics from the NCS and move towards a problem solving conception of the subject. Adopting such an approach to the teaching and learning of mathematics will in turn contribute to the intended curriculum being properly implemented and attained through the quality of learners coming out of the education system.

Practice	Example
Learners engage in solving contextual problems related to their lives that require them to interpret a problem and then find a suitable mathematical solution.	Learners are asked to work out which bus service is the cheapest given the fares they charge and the distance they want to travel.
Learners engage in solving problems of a purely mathematical nature, which require higher order thinking and application of knowledge (non-routine problems).	Learners are required to draw a graph; they have not yet been given a specific technique on how to draw (for example a parabola), but have learnt to use the table method to draw straight-line graphs.
Learners are given opportunities to negotiate meaning.	Learners discuss their understanding of concepts and strategies for solving problems with each other and the educator.
Learners are shown and required to represent situations in various but equivalent ways (mathematical modelling).	Learners represent data using a graph, a table and a formula to represent the same data.
Learners individually do mathematical investigations in class, guided by the educator where necessary.	Each learner is given a paper containing the mathematical problem (for instance to find the number of prime numbers less than 50) that needs to be investigated and the solution needs to be written up. Learners work independently.
Learners work together as a group/team to investigate or solve a mathematical problem.	A group is given the task of working together to solve a problem that requires them investigating patterns and working through data to make conjectures and find a formula for the pattern.
Learners do drill and practice exercises to consolidate the learning of concepts and to master various skills.	Completing an exercise requiring routine procedures.
Learners are given opportunities to see the interrelatedness of the mathematics and to see how the different outcomes are related and connected.	While learners work through geometry problems, they are encouraged to make use of algebra.
Learners are required to pose problems for their educator and peer learners.	Learners are asked to make up an algebraic word problem (for which they also know the solution) for the person sitting next to them to solve.

Overview of topics

Summary of topics and their relevance:

1. Functions – linear, quadratic, exponential, rational	Relevance
Relationships between variables in terms of graphical, verbal and symbolic representations of functions (tables, graphs, words and formulae). Generating graphs and generalising effects of parameters of vertical shifts and stretches and reflections about the x-axis. Problem solving and graph work involving prescribed functions.	Functions form a core part of learners' mathematical understanding and reasoning processes in algebra. This is also an excellent opportunity for contextual mathematical modelling questions.

2. Number Patterns, Sequences and Series	Relevance
Number patterns with constant difference.	Much of mathematics revolves around the identification of patterns.

3. Finance, Growth and Decay	Relevance
Use simple and compound growth formulae. Implications of fluctuating exchange rates.	The mathematics of finance is very relevant to daily and long-term financial decisions learners will need to make in terms of investing, taking loans, saving and understanding exchange rates and their influence more globally.

4. Algebra	Relevance
Understand that real numbers can be irrational or rational. Simplify expressions using the laws of exponents for rational exponents. Identifying and converting forms of rational numbers. Working with simple surds that are not rational. Working with laws of integral exponents. Establish between which two integers a simple surd lies. Appropriately rounding off real numbers. Manipulating and simplifying algebraic expressions (including multiplication and factorisation). Solving linear, quadratic, literal and exponential equations. Solving linear inequalities in one and two variables algebraically and graphically.	Algebra provides the basis for mathematics learners to move from numerical calculations to generalising operations, simplifying expressions, solving equations and using graphs and inequalities in solving contextual problems.

5. Differential Calculus	**Relevance**
Investigate average rate of change between two independent values of a function.	The central aspect of rate of change to differential calculus is a basis to further understanding of limits, gradients and calculations and formulae necessary for work in engineering fields, e.g. designing roads, bridges etc.

6. Probability	**Relevance**
Compare relative frequency and theoretical probability. Use Venn diagrams to solve probability problems. Mutually exclusive and complementary events. Identity for any two events A and B.	This topic is helpful in developing good logical reasoning in learners and for educating them in terms of real-life issues such as gambling and the possible pitfalls thereof.

7. Euclidean Geometry and Measurement	**Relevance**
Investigate, form and try to prove conjectures about properties of special triangles, quadrilaterals and other polygons. Disprove false conjectures using counter-examples. Investigate alternative definitions of various polygons. Solve problems involving surface area and volumes of solids and combinations thereof.	The thinking processes and mathematical skills of proving conjectures and identifying false conjectures is more the relevance here than the actual content studied. The surface area and volume content studied in real-life contexts of designing kitchens, tiling and painting rooms, designing packages, etc. is relevant to the current and future lives of learners.

8. Trigonometry	Relevance
Definitions of trig functions. Derive values for special angles. Take note of names for reciprocal functions. Solve problems in 2 dimensions. Extend definition of basic trig functions to all four quadrants and know graphs of these functions. Investigate and know the effects of a and q on the graphs of basic trig functions. Solve problems involving surface area and volumes of solids and combinations thereof.	Trigonometry has several uses within society, including within navigation, music, geographical locations and building design and construction.

9. Analytical Geometry	Relevance
Represent geometric figures on a Cartesian coordinate system. For any two points, derive and apply formula for calculating distance, gradient of line segment and coordinates of mid-point.	This section provides a further application point for learners' algebraic and trigonometric interaction with the Cartesian plane. Artists and design and layout industries often draw on the content and thought processes of this mathematical topic.

10. Statistics	Relevance
Collect, organise and interpret univariate numerical data to determine mean, median, mode, percentiles, quartiles, deciles, interquartile and semi-interquartile range. Identify possible sources of bias and errors in measurements.	Citizens are daily confronted with interpreting data presented from the media. Often this data may be biased or misrepresented within a certain context. In any type of research, data collection and handling is a core feature. This topic also educates learners to become more socially and politically educated with regards to the media.

Mathematics educators also need to ensure that the following important specific aims and general principles are applied in mathematics activities across all grades:

- Calculators should only be used to perform standard numerical computations and verify calculations done by hand.
- Real-life problems should be incorporated into all sections to keep mathematical modelling as an important focal point of the curriculum.
- Investigations give learners the opportunity to develop their ability to be more methodical, to generalise and to make and justify and/or prove conjectures.
- Appropriate approximation and rounding skills should be taught and continuously included and encouraged in activities.
- The history of mathematics should be incorporated into projects and tasks where possible, to illustrate the human aspect and developing nature of mathematics.
- Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues where possible.
- Conceptual understanding of when and why should also feature in problem types.
- Mixed ability teaching requires educators to challenge able learners and provide remedial support where necessary.
- Misconceptions exposed by assessment need to be dealt with and rectified by questions designed by educators.
- Problem solving and cognitive development should be central to all mathematics teaching and learning so that learners can apply the knowledge effectively.

Allocation of teaching time

Time allocation for Mathematics per week: 4 hours and 30 minutes e.g. six forty-five minute periods per week.

Term	**Topic**	**No. of weeks**
Term 1	Algebraic expressions	3
	Exponents	2
	Number patterns	1
	Equations and inequalities	2
	Trigonometry	3
Term 2	Functions	4
	Trigonometric functions	1
	Euclidean Geometry	3
	MID-YEAR EXAMS	3
Term 3	Analytical Geometry	2
	Finance and growth	2
	Statistics	2
	Trigonometry	2
	Euclidean Geometry	1
	Measurement	1
Term 4	Probability	2
	Revision	4
	EXAMS	3

Please see page 18 of the Curriculum and Assessment Policy Statement for the sequencing and pacing of topics.

Assessment

"Educator assessment is part of everyday teaching and learning in the classroom. Educators discuss with learners, guide their work, ask and answer questions, observe, help, encourage and challenge. In addition, they mark and review written and other kinds of work. Through these activities they are continually finding out about their learners' capabilities and achievements. This knowledge then informs plans for future work. It is this continuous process that makes up educator assessment. It should not be seen as a separate activity necessarily requiring the use of extra tasks or tests."

As the quote above suggests, assessment should be incorporated as part of the classroom practice, rather than as a separate activity. Research during the past ten years indicates that learners get a sense of what they do and do not know, what they might do about this and how they feel about it, from frequent and regular classroom assessment and educator feedback. The educator's perceptions of and approach to assessment (both formal and informal assessment) can have an influence on the classroom culture that is created with regard to the learners' expectations of and performance in assessment tasks. Literature on classroom assessment distinguishes between two different purposes of assessment; assessment of learning and assessment for learning.

Assessment of learning tends to be a more formal assessment and assesses how much learners have learnt or understood at a particular point in the annual teaching plan. The NCS provides comprehensive guidelines on the types of and amount of formal assessment that needs to take place within the teaching year to make up the school-based assessment mark. The school-based assessment mark contributes 25% of the final percentage of a learner's promotion mark, while the end-of-year examination constitutes the other 75% of the annual promotion mark. Learners are expected to have 7 formal assessment tasks for their school-based assessment mark. The number of tasks and their weighting in the Grade 10 Mathematics curriculum is summarised below:

		Tasks	**Weight (%)**
School-Based Assessment	Term 1	Test Project/Investigation	10 20
	Term 2	Assignment/Test Examination	10 30
	Term 3	Test Test	10 10
	Term 4	Test	10
School-Based Assessment Mark			100
School-Based Assessment Mark (as a % of Promotion Mark)			25 %
End-of-Year Examination			75 %
Promotion Mark			100 %

The following provides a brief explanation of each of the assessment tasks included in the assessment programme above.

Tests

All mathematics educators are familiar with this form of formal assessment. Tests include a variety of items/questions covering the topics that have been taught prior to the test. The new NCS also stipulates that mathematics tests should include questions that cover the following four types of cognitive levels in the stipulated weightings:

Cognitive Levels	Description	Weighting (%)
Knowledge	Estimation and appropriate rounding of numbers. Proofs of prescribed theorems. Derivation of formulae. Straight recall. Identification and direct use of formula on information sheet (no changing of the subject). Use of mathematical facts. Appropriate use of mathematical vocabulary.	20
Routine Procedures	Perform well known procedures. Simple applications and calculations. Derivation from given information. Identification and use (including changing the subject) of correct formula. Questions generally similar to those done in class.	45
Complex Procedures	Problems involve complex calculations and/or higher reasoning. There is often not an obvious route to the solution. Problems need not be based on real world context. Could involve making significant connections between different representations. Require conceptual understanding.	25
Problem Solving	Unseen, non-routine problems (which are not necessarily difficult). Higher order understanding and processes are often involved. Might require the ability to break the problem down into its constituent parts.	10

The breakdown of the tests over the four terms is summarised from the NCS assessment programme as follows:
Term 1: One test of at least 50 marks, and one hour or two/three tests of at least 40 minutes each.
Term 2: Either one test (of at least 50 marks) or an assignment.
Term 3: Two tests, each of at least 50 marks and one hour.
Term 4: One test of at least 50 marks.

Projects/Investigations

Investigations and projects consist of open-ended questions that initiate and expand thought processes. Acquiring and developing problem-solving skills are an essential part of doing investigations and projects. These tasks provide learners with the opportunity to investigate, gather information, tabulate results, make conjectures and justify or prove these conjectures. Examples of investigations and projects and possible marking rubrics are provided in the next section on assessment support. The NCS assessment programme indicates that only one project or investigation (of at least 50 marks) should be included per year. Although the project/investigation is scheduled in the assessment programme for the first term, it could also be done in the second term.

Assignments

The NCS includes the following tasks as good examples of assignments:

- Open book test
- Translation task
- Error spotting and correction
- Shorter investigation
- Journal entry
- Mind-map (also known as a metacog)
- Olympiad (first round)
- Mathematics tutorial on an entire topic
- Mathematics tutorial on more complex/problem solving questions

The NCS assessment programme requires one assignment in term 2 (of at least 50 marks) which could also be a combination of some of the suggested examples above. More information on these suggested examples of assignments and possible rubrics are provided in the following section on assessment support.

Examinations

Educators are also all familiar with this summative form of assessment that is usually completed twice a year: mid-year examinations and end-of-year examinations. These are similar to the tests but cover a wider range of topics completed prior to each examination. The NCS stipulates that each examination should also cover the four cognitive levels according to their recommended weightings as summarised in the section above on tests. The following table summarises the requirements and information from the NCS for the two examinations.

Examination	**Marks**	**Breakdown**	**Content and Mark Distribution**
Mid-Year Exam	100 50 + 50	One paper: 2 hours **or** Two papers: each of 1 hour	Topics completed
End-of-Year Exam	100 +	Paper 1: 2 hours	Number patterns (±10) Algebraic expressions, equations and inequalities (±25) Functions (±35) Exponents (±10) Finance (±10) Probability (±10)
	100	Paper 2: 2 hours	Trigonometry (±45) Analytical geometry (±15) Euclidean geometry and measurement (±25) Statistics (±15)

In the annual teaching plan summary of the NCS in Mathematics for Grade 10, the pace setter section provides a detailed model of the suggested topics to be covered each week of each term and the accompanying formal assessment. Assessment **for** learning tends to be more informal and focuses on using assessment in and of daily classroom activities that can include:

- Marking homework
- Baseline assessments
- Diagnostic assessments
- Group work
- Class discussions
- Oral presentations
- Self-assessment
- Peer-assessment

These activities are expanded on in the next section on assessment support and suggested marking rubrics are provided. Where formal assessment tends to restrict the learner to written assessment tasks, the informal assessment is necessary to evaluate and encourage the progress of the learners in their verbal mathematical reasoning and communication skills. It also provides a less formal assessment environment that allows learners to openly and honestly assess themselves and each other, taking responsibility for their own learning, without the heavy weighting of the performance (or mark) component. The assessment for learning tasks should be included in the classroom activities at least once a week (as part of a lesson) to ensure that the educator is able to continuously evaluate the learners' understanding of the topics covered as well as the effectiveness, and identify any possible deficiencies in his or her own teaching of the topics.

Assessment support

A selection of explanations, examples and suggested marking rubrics for the assessment of learning (formal) and the assessment for learning (informal) forms of assessment discussed in the preceding section are provided in this section.

Baseline assessment

Baseline assessment is a means of establishing:

- What prior knowledge a learner possesses
- What the extent of knowledge is that they have regarding a specific learning area
- The level they demonstrate regarding various skills and applications
- The learner's level of understanding of various learning areas

It is helpful to educators in order to assist them in taking learners from their individual point of departure to a more advanced level and to thus make progress. This also helps avoid large "gaps" developing in the learners' knowledge as the learner moves through the education system. Outcomes-based education is a more learner-centered approach than we are used to in South Africa, and therefore the emphasis should now be on the level of each individual learner rather than that of the whole class.

The baseline assessments also act as a gauge to enable learners to take more responsibility for their own learning and to view their own progress. In the traditional assessment system, the weaker learners often drop from a 40% average in the first term to a 30% average in the fourth term due to an increase in workload, thus demonstrating no obvious progress. Baseline assessment, however, allows for an initial assigning of levels which can be improved upon as the learner progresses through a section of work and shows greater knowledge, understanding and skill in that area.

Diagnostic assessments

These are used to specifically find out if any learning difficulties or problems exist within a section of work in order to provide the learner with appropriate additional help and guidance. The assessment helps the educator and the learner identify problem areas, misunderstandings, misconceptions and incorrect use and interpretation of notation.

Some points to keep in mind:

- Try not to test too many concepts within one diagnostic assessment.
- Be selective in the type of questions you choose.
- Diagnostic assessments need to be designed with a certain structure in mind. As an educator, you should decide exactly what outcomes you will be assessing and structure the content of the assessment accordingly.
- The assessment is marked differently to other tests in that the mark is not the focus but rather the type of mistakes the learner has made.

An example of an understanding rubric for educators to record results is provided below:
0: indicates that the learner has not grasped the concept at all and that there appears to be a fundamental mathematical problem.
1: indicates that the learner has gained some idea of the content, but is not demonstrating an understanding of the notation and concept.
2: indicates evidence of some understanding by the learner but further consolidation is still required.
3:indicates clear evidence that the learner has understood the concept and is using the notation correctly.

Calculator worksheet - diagnostic skills assessment

1. Calculate:
   - **(a)** $242 + 63 =$ _______
   - **(b)** $2 - 36 \times (114 + 25) =$ _______
   - **(c)** $\sqrt{144 + 25} =$ _______
   - **(d)** $\sqrt[4]{729} =$ _______
   - **(e)** $-312 + 6 + 879 - 321 + 18\ 901 =$ _______
2. Calculate:
   - **(a)** $\frac{2}{7} + \frac{1}{3} =$ _______
   - **(b)** $2\frac{1}{5} - \frac{2}{9} =$ _______
   - **(c)** $-2\frac{5}{6} + \frac{3}{8} =$ _______
   - **(d)** $4 - \frac{3}{4} \times \frac{5}{7} =$ _______
   - **(e)** $\left(\frac{9}{10} - \frac{8}{9}\right) \div \frac{3}{5} =$ _______
   - **(f)** $2 \times \left(\frac{4}{5}\right)^2 - \left(\frac{19}{25}\right) =$ _______
   - **(g)** $\sqrt{\frac{9}{4} - \frac{4}{16}} =$ _______

Self-Assesment Rubric:
Name: ________________

Question	**Answer**	√	**X**	**If X, write down sequence of keys pressed**
1a)				
1b)				
1c)				
1d)				
1e)				
Subtotal				
2a)				
2b)				
2c)				
2d)				
2e)				
2f)				
2g)				
Subtotal				
Total				

Educator Assessment Rubric:

Type of Skill	**Competent**	**Needs Practice**	**Problem**
Raising to a Power			
Finding a Root			
Calculations with Fractions			
Brackets and Order of Operations			
Estimation and Mental Control			

Guidelines for Calculator Skills Assessment:

Type of Skill	Sub-Division	Questions
Raising to a Power	Squaring and cubing Higher order powers	1a, 2f 1b
Finding a Root	Square and cube roots Higher order roots	1c, 2g 1d
Calculations with Fractions	Basic operations Mixed numbers Negative numbers Squaring fractions Square rooting fractions	2a, 2d 2b, 2c 1e, 2c 2f 2g
Brackets and Order of Operations	Correct use of brackets or order of operations	1b, 1c, 2e, 2f, 2g
Estimation and Mental Control	Overall	All

Suggested guideline to allocation of overall levels

Level 1

- Learner is able to do basic operations on calculator.
- Learner is able to do simple calculations involving fractions.
- Learner does not display sufficient mental estimation and control techniques.

Level 2

- Learner is able to do basic operations on calculator.
- Learner is able to square and cube whole numbers as well as find square and cube roots of numbers.
- Learner is able to do simple calculations involving fractions as well as correctly execute calculations involving mixed numbers.
- Learner displays some degree of mental estimation awareness.

Level 3

- Learner is able to do basic operations on calculator.
- Learner is able to square and cube rational numbers as well as find square and cube roots of numbers.
- Learner is also able to calculate higher order powers and roots.
- Learner is able to do simple calculations involving fractions as well as correctly execute calculations involving mixed numbers.
- Learner works correctly with negative numbers.
- Learner is able to use brackets in certain calculations but has still not fully understood the order of operations that the calculator has been programmed to execute, hence the need for brackets.
- Learner is able to identify possible errors and problems in their calculations but needs assistance solving the problem.

Level 4

- Learner is able to do basic operations on calculator.
- Learner is able to square and cube rational numbers as well as find square and cube roots.
- Learner is also able to calculate higher order powers and roots.
- Learner is able to do simple calculations involving fractions as well as correctly execute calculations involving mixed numbers.
- Learner works correctly with negative numbers.
- Learner is able to work with brackets correctly and understands the need and use of brackets and the "= key" in certain calculations due to the nature of a scientific calculator.
- Learner is able to identify possible errors and problems in their calculations and to find solutions to these in order to arrive at a "more viable" answer.

Other short diagnostic tests

These are short tests that assess small quantities of recall knowledge and application ability on a day-to-day basis. Such tests could include questions on one or a combination of the following:

- Definitions
- Theorems
- Riders (geometry)
- Formulae
- Applications
- Combination questions

Here is a selection of model questions that can be used at Grade 10 level to make up short diagnostic tests. They can be marked according to a memorandum drawn up by the educator.

Geometry

1. Points $A(-5;-3)$, $B(-1;2)$ and $C(9;-6)$ are the vertices of ΔABC.

   (a) Calculate the gradients of AB and BC and hence show that angle ABC is equal to $90°$.

   (5)

   (b) State the distance formula and use it to calculate the lengths of the sides AB, BC and AC of ΔABC. (Leave your answers in surd form).

   (5)

Algebra

1. Write down the formal definition of an exponent as well as the exponent laws for integral exponents.

   (6)

2. Simplify: $\frac{2x^4y^8z^3}{4xy} \times \frac{x^7}{y^3z^0}$

   (4)

Trigonometry

1. A jet leaves an airport and travels 578 km in a direction of $50°$ E of N. The pilot then changes direction and travels 321 km $10°$ W of N.
   (a) How far away from the airport is the jet? (To the nearest kilometre)
   (5)
   (b) Determine the jet's bearing from the airport.
   (5)

Exercises

This entails any work from the textbook or other source that is given to the learner, by the educator, to complete either in class or at home. Educators should encourage learners not to copy each other's work and be vigilant when controlling this work. It is suggested that such work be marked/controlled by a check list (below) to speed up the process for the educator.

The marks obtained by the learner for a specific piece of work need not be based on correct and/or incorrect answers but preferably on the following:

- the effort of the learner to produce answers.
- the quality of the corrections of work that was previously incorrect.
- the ability of the learner to explain the content of some selected examples (whether in writing or orally).

The following rubric can be used to assess exercises done in class or as homework:

Criteria	**Performance Indicators**		
Work Done	2 All the work	1 Partially completed	0 No work done
Work Neatly Done	2 Work neatly done	1 Some work not neatly done	0 Messy and muddled
Corrections Done	2 All corrections done consistently	1 At least half of the corrections done	0 No corrections done
Correct Mathematical Method	2 Consistently	1 Sometimes	0 Never
Understanding of Mathematical Techniques and Processes	2 Can explain concepts and processes precisely	1 Explanations are ambiguous or not focused	0 Explanations are confusing or irrelevant

Journal entries

A journal entry is an attempt by a learner to express in the written word what is happening in Mathematics. It is important to be able to articulate a mathematical problem, and its solution in the written word.

This can be done in a number of different ways:

- Today in Maths we learnt ________________
- Write a letter to a friend, who has been sick, explaining what was done in class today.
- Explain the thought process behind trying to solve a particular maths problem, e.g. sketch the graph of $y = x^2 - 2x^2 + 1$ and explain how to sketch such a graph.
- Give a solution to a problem, decide whether it is correct and if not, explain the possible difficulties experienced by the person who wrote the incorrect solution.

A journal is an invaluable tool that enables the educator to identify any mathematical misconceptions of the learners. The marking of this kind of exercise can be seen as subjective but a marking rubric can simplify the task. The following rubric can be used to mark journal entries. The learners must be given the marking rubric before the task is done.

Task	**Competent (2 Marks)**	**Still developing (1 Mark)**	**Not yet developed (1 Mark)**
Completion in time limit?			
Correctness of the explanation?			
Correct and relevant use of mathematical language?			
Is the mathematics correct?			
Has the concept been interpreted correctly?			

Translations

Translations assess the learner's ability to translate from words into mathematical notation or to give an explanation of mathematical concepts in words. Often when learners can use mathematical language and notation correctly, they demonstrate a greater understanding of the concepts.

For example:
Write the letter of the correct expression next to the matching number:

x increased by 10	a)	xy
The product of x and y	b)	$x - 2$
The sum of a certain number and double that number	c)	x^2
Half of a certain number multiplied by itself	d)	$\frac{1}{2} \times 2$
Two less than x	e)	$x + 2x$
A certain number multiplied by itself	f)	$x + 10$

Group work

One of the principles in the NCS is to produce learners who are able to work effectively within a group. Learners generally find this difficult to do. Learners need to be encouraged to work within small groups. Very often it is while learning under peer assistance that a better understanding of concepts and processes is reached. Clever learners usually battle with this sort of task, and yet it is important that they learn how to assist and communicate effectively with other learners.

Mind maps or metacogs

A metacog or "mind map" is a useful tool. It helps to associate ideas and make connections that would otherwise be too unrelated to be linked. A metacog can be used at the beginning or end of a section of work in order to give learners an overall perspective of the work covered, or as a way of recalling a section already completed. It must be emphasised that it is not a summary. Whichever way you use it, it is a way in which a learner is given the opportunity of doing research in a particular field and can show that he/she has an understanding of the required section.

This is an open book form of assessment and learners may use any material they feel will assist them. It is suggested that this activity be practised, using other topics, before a test metacog is submitted for portfolio assessment purposes.

On completion of the metacog, learners must be able to answer insightful questions on the metacog. This is what sets it apart from being just a summary of a section of work. Learners must refer to their metacog when answering the questions, but may not refer to any reference material. Below are some guidelines to give to learners to adhere to when constructing a metacog as well as two examples to help you get learners started. A marking rubric is also provided. This should be made available to learners before they start constructing their metacogs. On the next page is a model question for a metacog, accompanied by some sample questions that can be asked within the context of doing a metacog about analytical geometry.

A basic metacog is drawn in the following way:

- Write the title/topic of the subject in the centre of the page and draw a circle around it.
- For the first main heading of the subject, draw a line out from the circle in any direction, and write the heading above or below the line.
- For sub-headings of the main heading, draw lines out from the first line for each subheading and label each one.
- For individual facts, draw lines out from the appropriate heading line.

Metacogs are one's own property. Once a person understands how to assemble the basic structure they can develop their own coding and conventions to take things further, for example to show linkages between facts. The following suggestions may assist educators and learners to enhance the effectiveness of their metacogs:

- Use single words or simple phrases for information. Excess words just clutter the metacog and take extra time to write down.
- Print words – joined up or indistinct writing can be more difficult to read and less attractive to look at.
- Use colour to separate different ideas – this will help your mind separate ideas where it is necessary, and helps visualisation of the metacog for easy recall. Colour also helps to show organisation.
- Use symbols and images where applicable. If a symbol means something to you, and conveys more information than words, use it. Pictures also help you to remember information.
- Use shapes, circles and boundaries to connect information – these are additional tools to help show the grouping of information.

Use the concept of analytical geometry as your topic and construct a mind map (or metacog) containing all the information (including terminology, definitions, formulae and examples) that you know about the topic of analytical geometry.

Possible questions to ask the learner on completion of their metacog:

- Briefly explain to me what the mathematics topic of analytical geometry entails.
- Identify and explain the distance formula, the derivation and use thereof for me on your metacog.
- How does the calculation of gradient in analytical geometry differ (or not) from the approach used to calculate gradient in working with functions?

A suggested simple rubric for marking a metacog:

Task	**Competent (2 Marks)**	**Still Developing (1 Mark)**	**Not Yet Developed 1 Mark)**
Completion in Time Limit			
Main Headings			
Correct Theory (Formulae, Definitions, Terminology etc.)			
Explanation			
"Readability"			

10 marks for the questions, which are marked using the following scale:
0 - no attempt or a totally incorrect attempt has been made
1 - a correct attempt was made, but the learner did not get the correct answer

2 - a correct attempt was made and the answer is correct

Investigations

Investigations consist of open-ended questions that initiate and expand thought processes. Acquiring and developing problem-solving skills are an essential part of doing investigations.

It is suggested that 2 – 3 hours be allowed for this task. During the first 30 – 45 minutes learners could be encouraged to talk about the problem, clarify points of confusion, and discuss initial conjectures with others. The final written-up version should be done individually though and should be approximately four pages.

Assessing investigations may include feedback/ presentations from groups or individuals on the results keeping the following in mind:

- following of a logical sequence in solving the problems
- pre-knowledge required to solve the problem
- correct usage of mathematical language and notation
- purposefulness of solution
- quality of the written and oral presentation

Some examples of suggested marking rubrics are included on the next few pages, followed by a selection of topics for possible investigations.

The following guidelines should be provided to learners before they begin an investigation:

General Instructions Provided to Learners

- You may choose any one of the projects/investigations given (see model question on investigations)
- You should follow the instructions that accompany each task as these describe the way in which the final product must be presented.
- You may discuss the problem in groups to clarify issues, but each individual must write-up their own version.
- Copying from fellow learners will cause the task to be disqualified.
- Your educator is a resource to you, and though they will not provide you with answers / solutions, they may be approached for hints.

The Presentation The investigation is to be handed in on the due date, indicated to you by your educator. It should have as a minimum:

- A description of the problem.
- A discussion of the way you set about dealing with the problem.
- A description of the final result with an appropriate justification of its validity.
- Some personal reflections that include mathematical or other lessons learnt, as well as the feelings experienced whilst engaging in the problem.
- The written-up version should be attractively and neatly presented on about four A4 pages.
- Whilst the use of technology is encouraged in the presentation, the mathematical content and processes must remain the major focus.

Below are some examples of possible rubrics to use when marking investigations:

Level of Performance	Criteria
4	• Contains a complete response. • Clear, coherent, unambiguous and elegant explanation. • Includes clear and simple diagrams where appropriate. • Shows understanding of the question's mathematical ideas and processes. • Identifies all the important elements of the question. • Includes examples and counter examples. • Gives strong supporting arguments. • Goes beyond the requirements of the problem.
3	• Contains a complete response. • Explanation less elegant, less complete. • Shows understanding of the question's mathematical ideas and processes. • Identifies all the important elements of the question. • Does not go beyond the requirements of the problem.
2	• Contains an incomplete response. • Explanation is not logical and clear. • Shows some understanding of the question's mathematical ideas and processes. • Identifies some of the important elements of the question. • Presents arguments, but incomplete. • Includes diagrams, but inappropriate or unclear.
1	• Contains an incomplete response. • Omits significant parts or all of the question and response. • Contains major errors. • Uses inappropriate strategies.
0	• No visible response or attempt

Orals

An oral assessment involves the learner explaining to the class as a whole, a group or the educator his or her understanding of a concept, a problem or answering specific questions. The focus here is on the correct use of mathematical language by the learner and the conciseness and logical progression of their explanation as well as their communication skills.

Orals can be done in a number of ways:

- A learner explains the solution of a homework problem chosen by the educator.
- The educator asks the learner a specific question or set of questions to ascertain that the learner understands, and assesses the learner on their explanation.
- The educator observes a group of learners interacting and assesses the learners on their contributions and explanations within the group.
- A group is given a mark as a whole, according to the answer given to a question by any member of a group.

An example of a marking rubric for an oral:
1 - the learner has understood the question and attempts to answer it
2 - the learner uses correct mathematical language
2 - the explanation of the learner follows a logical progression
2 - the learner's explanation is concise and accurate
2 - the learner shows an understanding of the concept being explained
1 - the learner demonstrates good communication skills
Maximum mark = 10

An example of a peer-assessment rubric for an oral:
My name: ___________________
Name of person I am assessing: ___________________

Criteria	**Mark Awarded**	**Maximum Mark**
Correct Answer		2
Clarity of Explanation		3
Correctness of Explanation		3
Evidence of Understanding		2
Total		10

Chapter Contexts

Algebraic expressions

Algebra provides the basis for mathematics learners to move from numerical calculations to generalising operations, simplifying expressions, solving equations and using graphs and inequalities in solving contextual problems. Being able to multiply out and factorise are core skills in the process of simplifying expressions and solving equations in mathematics. Identifying irrational numbers and knowing their estimated position on a number line or graph is an important part of any mathematical calculations and processes that move beyond the basic number system of whole numbers and integers. Rounding off irrational numbers (such as the value of π) when needed, allows mathematics learners to work more efficiently with numbers that would otherwise be difficult to "pin down", use and comprehend.

Once learners have grasped the basic number system of whole numbers and integers, it is vital that their understanding of the numbers between integers is also expanded. This incorporates their dealing with fractions, decimals and surds which form a central part of most mathematical calculations in real-life contextual issues. Estimation is an extremely important component within mathematics. It allows learners to work with a calculator or present possible solutions while still being able to gauge how accurate and realistic their answers may be, which is relevant for other subjects too.

Equations and inequalities

If learners are to later work competently with functions and the graphing and interpretation thereof, their understanding and skills in solving equations and inequalities will need to be developed.

Exponents

Exponential notation is a central part of mathematics in numerical calculations as well as algebraic reasoning and simplification. It is also a necessary component for learners to understand and appreciate certain financial concepts such as compound interest and growth and decay.

Number patterns

Much of mathematics revolves around the identification of patterns. In earlier grades learners saw patterns in the form of pictures and numbers. In this chapter we look at the mathematics of patterns. Patterns are repetitive sequences and can be found in nature, shapes, events, sets of numbers and almost everywhere you care to look. For example, seeds in a sunflower, snowflakes, geometric designs on quilts or tiles, the number sequence $0;\ 4;\ 8;\ 12;\ 16;\ldots$

Functions

Functions form a core part of learners' mathematical understanding and reasoning processes in algebra. This is also an excellent opportunity for contextual mathematical modelling questions. Functions are mathematical building blocks for designing machines, predicting natural disasters, curing diseases, understanding world economies and for keeping aeroplanes in the air. A useful advantage of functions is that they allow us to visualise relationships in terms of a graph. Functions are much easier to read and interpret than lists of numbers. In addition to their use in the problems facing humanity, functions also appear on a day-to-day level, so they are worth learning about. The value of a function is always dependent on one or more variables, like time, distance or a more abstract quantity.

Finance and Growth

The mathematics of finance is very relevant to daily and long-term financial decisions learners will need to take in terms of investing, taking loans, saving and understanding exchange rates and their influence more globally.

Trigonometry

There are many applications of trigonometry. Of particular value is the technique of triangulation, which is used in astronomy to measure the distances to nearby stars, in geography to measure distances between landmarks, and in satellite navigation systems. GPS (the global positioning system) would not be possible without trigonometry. Other fields which make use of trigonometry include acoustics, optics, analysis of financial markets, electronics, probability theory, statistics, biology, medical imaging (CAT scans and ultrasound), chemistry, cryptology, meteorology, oceanography, land surveying, architecture, phonetics, engineering, computer graphics and game development.

Analytical geometry

This section provides a further application point for learners' algebraic and trigonometric interaction with the Cartesian plane. Artists and design and layout industries often draw on the content and thought processes of this mathematical topic.

Statistics

Citizens are daily confronted with interpreting data presented from the media. Often this data may be biased or misrepresented within a certain context. In any type of research, data collection and handling is a core feature. This topic also educates learners to become more socially and politically educated with regards to the media.

Probability

This topic is helpful in developing good logical reasoning in learners and for educating them in terms of real-life issues such as gambling and the possible pitfalls thereof. We use probability to describe uncertain events: when you accidentally drop a slice of bread, you don't know if it's going to fall with the buttered side facing upwards or downwards. When your favourite sports team plays a game, you don't know whether they will win or not. When the weatherman says that there is a 40% chance of rain tomorrow, you may or may not end up getting wet. Uncertainty presents itself to some degree in every event that occurs around us and in every decision that we make.

Euclidean geometry

The thinking processes and mathematical skills of proving conjectures and identifying false conjectures is more the relevance here than the actual content studied. The surface area and volume content studied in real-life contexts of designing kitchens, tiling and painting rooms, designing packages, etc. is relevant to the current and future lives of learners. Euclidean geometry deals with space and shape using a system of logical deductions.

Measurement

This chapter revises the volume and surface areas of three-dimensional objects, otherwise known as solids. The chapter covers the volume and surface area of prisms and cylinders, Many exercises cover finding the surface area and volume of polygons, prisms, pyramids, cones and spheres, as well as a complex object. The effect on volume and surface area when multiplying a dimension of a factor of k is also explored.

Exercise solutions

This chapter includes the solutions to the exercises covered in each chapter of the book.

Part II

Solutions

Algebraic expressions 1

Exercise 1 - 1

1. State whether the following numbers are rational or irrational. If the number is rational, state whether it is a natural number, whole number or an integer:
   (a) $-\frac{1}{3}$
   (b) $0{,}651268962154862\ldots$
   (c) $\frac{\sqrt{9}}{3}$
   (d) π^2
2. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?
   (a) $\frac{5}{6}$
   (b) $\frac{a}{3}$
   (c) $\frac{-2}{b}$
   (d) $\frac{1}{c}$
3. For which of the following values of a is $\frac{a}{14}$ rational or irrational?
   (a) 1
   (b) -10
   (c) $\sqrt{2}$
   (d) $2{,}1$
4. Write the following as fractions:
   (a) $0{,}1$
   (b) $0{,}12$
   (c) $0{,}58$
   (d) $0{,}2589$
5. Write the following using the recurring decimal notation:
   (a) $0{,}11111111\ldots$
   (b) $0{,}1212121212\ldots$
   (c) $0{,}123123123123\ldots$
   (d) $0{,}11414541454145\ldots$
6. Write the following in decimal form, using the recurring decimal notation:

(a) $\frac{2}{3}$

(b) $1\frac{3}{11}$

(c) $4\frac{5}{6}$

(d) $2\frac{1}{9}$

7. Write the following decimals in fractional form:

(a) $0{,}\dot{5}$

(b) $0{,}6\dot{3}$

(c) $5{,}\overline{31}$

Solutions to Exercise 1 - 1

1. (a) Rational, integer
   (b) Irrational
   (c) Rational, integer, whole, natural number
   (d) Irrational

2. (a) Rational
   (b) Rational
   (c) Rational
   (d) Irrational

3. (a) Rational
   (b) Rational
   (c) Irrational
   (d) Rational

4. (a) $0{,}1 = \frac{1}{10}$
   (b) $0{,}12 = \frac{12}{100} = \frac{3}{25}$
   (c) $0{,}58 = \frac{58}{100} = \frac{29}{50}$
   (d) $0{,}2589 = \frac{2\ 589}{10\ 000}$

5. (a) $0{,}\dot{1}$
   (b) $0{,}\overline{12}$
   (c) $0{,}\overline{123}$
   (d) $0{,}11\overline{4145}$

6. (a) $\frac{2}{3} = 2\left(\frac{1}{3}\right)$
   $= 2(0{,}333333\ldots)$
   $= 0{,}666666\ldots$

$= 0{,}\dot{6}$

(b) $1\frac{3}{11} = 1 + 3\left(\frac{1}{11}\right)$
$= 1 + 3(0{,}090909\ldots)$
$= 1 + 0{,}27272727\ldots$
$= 1{,}\overline{27}$

(c) $4\frac{5}{6} = 4 + 5\left(\frac{1}{6}\right)$
$= 4 + 5(0{,}1666666\ldots)$
$= 4 + 0{,}833333\ldots$
$= 4{,}8\dot{3}$

(d) $2\frac{1}{9} = 2 + 0{,}1111111\ldots$
$= 2{,}\dot{1}$

7. **(a)** $x = 0{,}55555$ and
$10x = 5{,}55555$
$\therefore 10x - x = 9x$
$= 5$
$\therefore x = \frac{5}{9}$

(b) $10x = 6{,}3333$ and
$100x = 63{,}3333$
$\therefore 100x - 10x = 90x$
$= 57$
$\therefore x = \frac{57}{90}$

(c) $x = 5{,}313131$ and
$100x = 531{,}313131$
$\therefore 100x - x = 99x =$
526
$\therefore x = \frac{526}{99}$

Exercise 1 - 2

Round off the following to 3 decimal places:

1. $12{,}56637061\ldots$
2. $3{,}31662479\ldots$
3. $0{,}26666666\ldots$
4. $1{,}912931183\ldots$
5. $6{,}32455532\ldots$
6. $0{,}05555555\ldots$

Solutions to Exercise 1 - 2

1. 12,5666
2. 3,317
3. 0,267
4. 1,913
5. 6,325
6. 0,056

Exercise 1 - 3

Determine between which two consecutive integers the following numbers lie, without using a calculator:

1. $\sqrt{18}$
2. $\sqrt{29}$
3. $\sqrt[3]{5}$
4. $\sqrt[3]{79}$

Solutions to Exercise 1 - 3

1. 4 and 5 ($4^2 = 16$ and $5^2 = 25$)
2. 5 and 6 ($5^2 = 25$ and $6^2 = 36$)
3. 1 and 2 ($1^3 = 1$ and $2^3 = 8$)
4. 4 and 5 ($4^3 = 64$ and $5^3 = 125$)

Exercise 1 - 4

Expand the following products:

1. $2y(y + 4)$
2. $(y + 5)(y + 2)$
3. $(2 - t)(1 - 2t)$
4. $(x - 4)(x + 4)$
5. $(2p + 9)(3p + 1)$
6. $(3k - 2)(k + 6)$
7. $(s + 6)^2$
8. $-(7 - x)(7 + x)$

9. $(3x-1)(3x+1)$
10. $(7k+2)(3-2k)$
11. $(1-4x)^2$
12. $(-3-y)(5-y)$
13. $(8-x)(8+x)$
14. $(9+x)^2$
15. $(-2y^2-4y+11)(5y-12)$
16. $(7y^2-6y-8)(-2y+2)$
17. $(10y+3)(-2y^2-11y+2)$
18. $(-12y-3)(12y^2-11y+3)$
19. $(-10)(2y^2+8y+3)$
20. $(2y^6+3y^5)(-5y-12)$
21. $(-7y+11)(-12y+3)$
22. $(7y+3)(7y^2+3y+10)$
23. $9(8y^2-2y+3)$
24. $(-6y^4+11y^2+3y)(y+4)(y-4)$

Solutions to Exercise 1 - 4

1. $2y(y+4)=2y^2+8y$
2. $(y+5)(y+2)$
$=y^2+2y+5y+10$
$=y^2+7y+10$
3. $(2-t)(1-2t)$
$=2+4t-t+2t^2$
$=2+3t+2t^2$
4. $(x-4)(x+4)$
$=x^2+4x-4x-16$
$=x^2-16$
5. $(2p+9)(3p+1)$
$=6p^2+2p+27p+9$
$=6p^2+29p+9$
6. $(3k-2)(k+6)$
$=3k^2+18k-2k-12$
$=3k^2+16k-12$
7. $(s+6)^2$
$=s^2+6s+6s+36$
$=s^2+12s+36$
8. $-(7-x)(7+x)$
$=-(49+7x-7x-x^2)$
$=-(49-x^2)$
$=-49+x^2$
9. $(3x-1)(3x+1)$
$=9x^2+3x-3x-1$
$=9x^2-1$
10. $(7k+2)(3-2k)$
$=21k-14k^2+6-4k$
$=14k^2+17k+6$
11. $(1-4x)^2$
$=(1-4x)(1-4x)$
$=1-4x-4x+16x^2$
$=1-8x+16x^2$
12. $(-3-y)(5-y)$
$=-15+3y-5y+y^2$
$=y^2-2y-15$
13. $(8-x)(8+x)$
$=16+8x-8x-x^2$
$=16-x^2$
14. $(9+x)^2$
$=(9+x)(9+x)$
$=81+9x+9x+x^2$
$=81+18x+x^2$
15. $(-2y^2-4y+11)(5y-12)$
$=-10y^3+24y^2-20y^2+48y+55y-$
132
$=-10y^3+4y^2+103y-132$
16. $(7y^2-6y-8)(-2y+2)$
$=-14y^3+14y^2+12y^2-12y+16y-16$
$=-14y^3+26y^2+4y-16$

17. $(10y + 3)(-2y^2 - 11y + 2)$
$= -20y^3 - 110y^2 + 20y - 6y^2 - 33y + 6$
$= -20y^3 - 116y^2 - 13y + 6$

18. $(-12y - 3)(12y^2 - 11y + 3)$
$= -144y^3 + 132y^2 - 36y - 36y^2 + 33y - 9$
$= -144y^3 + 96y^2 - 3y - 9$

19. $(-10)(2y^2 + 8y + 3) = -20y^2 - 80y - 30$

20. $(2y^6 + 3y^5)(-5y - 12)$
$= -10y^7 - 24y^6 - 15y^6 - 36y^5$
$= -10y^7 - 39y^6 - 36y^5$

21. $(-7y + 11)(-12y + 3)$
$= 84y^2 - 21y - 132y + 33$
$= 84y^2 - 153y + 33$

22. $(7y + 3)(7y^2 + 3y + 10)$
$= 49y^3 + 21y^2 + 70y + 21y^2 + 9y + 10$
$= 49y^3 + 42y^2 + 79y + 10$

23. $(9)(8y^2 - 2y + 3) = 72y^2 - 18y + 27$

24. $(-6y^4 + 11y^2 + 3y)(10y + 4)(4y - 4)$
$= (-6y^4 + 11y^2 + 3y)(y^2 + 16)$
$= -6y^6 - 96y^4 + 11y^4 + 176y^2 + 3y^3 + 48y$
$= -6y^6 - 85y^4 + 3y^3 + 176y^2 + 48y$

Exercise 1 - 5

Factorise:

1. $2l + 2w$
2. $12x + 32y$
3. $6x^2 + 2x + 10x^3$
4. $2xy^2 + xy^2z + 3xy$
5. $-2ab^2 - 4a^2b$
6. $7a + 4$
7. $20a - 10$
8. $18ab - 3bc$
9. $12kj + 18kq$
10. $16k^2 - 4$
11. $3a^2 + 6a - 18$
12. $-12a + 24a^3$
13. $-2ab - 8a$
14. $24kj - 16k^2j$
15. $-a^2b - b^2a$
16. $12k^2j + 24k^2j^2$
17. $72b^2q - 18b^3q^2$
18. $4(y - 3) + k(3 - y)$
19. $a^2(a - 1) - 25(a - 1)$
20. $bm(b + 4) - 6m(b + 4)$
21. $a^2(a + 7) + 9(a + 7)$
22. $3b(b - 4) - 7(4 - b)$
23. $a^2b^2c^2 - 1$

Solutions to Exercise 1 - 5

1. $2l + 2w = 2(l + w)$
2. $12x + 32y = 4(3x + 8y)$
3. $6x^2 + 2x + 10x^3 = 2x(3x + 1 + 5x^2)$
4. $2xy^2 + xy^2z + 3xy = xy(2y^2 + yz + 3)$

5. $-2ab^2 - 4a^2b = -2ab(b + a)$
6. $7a + 4 = 7a + 4$
7. $20a - 10 = 10(2a - 1)$
8. $18ab - 3bc = 3b(6a - c)$
9. $12kj + 18kq = 6k(2j + 3q)$
10. $16k^2 - 4 = (4k - 2)(4k + 2)$
11. $3a^2 + 6a - 1 = 3(a^2 + 2a - 9)$
12. $-12a + 24a^3 = 12a(2a^2 - 1)$
13. $-2ab - 8a = -2a(b + 4)$
14. $24kj - 16k^2j = 8kj(3 - 2k)$
15. $-a^2b - b^2a = -ab(a + b)$
16. $12k^2j + 24k^2j^2 = 12jk^2(2j + 1)$
17. $72b^2q - 18b^3q^2 = 18b^2q(4 - bq)$
18. $4(y-3)+k(3-y) = (3-y)(-4+k)$
19. $a^2(a - 1) - 25(a - 1)$
    $= (a - 1)(a^2 - 25)$
    $= (a - 1)(a - 5)(a + 5)$
20. $bm(b + 4) - 6m(b + 4)$
    $= (b + 4)(bm - 6m)$
    $= (b + 4)(m)(b - 6)$
21. $a^2(a+7)+9(a+7) = (a+7)(a^2+9)$
22. $3b(b-4) - 7(4-b) = (b-4)(3b+7)$
23. $a^2b^2c^2 - 1 = (abc - 1)(abc + 1)$

Exercise 1 - 6

Factorise the following:

1. $6x + a + 2ax + 3$
2. $x^2 - 6x + 5x - 30$
3. $5x + 10y - ax - 2ay$
4. $a^2 - 2a - ax + 2x$
5. $5xy - 3y + 10x - 6$
6. $ab - a^2 - a + b$

Solutions to Exercise 1 - 6

1. $6x + a + 2ax + 3$
   $= 3(2x + 1) + a(2x + 1)$
   $= (3 + a)(2x + 1)$
2. $x^2 - 6x + 5x - 30$
   $= x(x - 6) + 5(x - 6)$
   $= (x + 5)(x - 6)$
3. $5x + 10y - ax - 2ay$
   $= 5(x + 2y) - a(x + 2y)$
   $= (5 - a)(x + 2y)$
4. $a^2 - 2a - ax + 2x$
   $= a(a - 2) - x(a - 2)$
   $= (a - x)(a - 2)$
5. $5xy - 3y + 10x - 6$
   $= y(5x - 3) + 2(5x - 3)$
   $= (y + 2)(5x - 3)$
6. $ab - a^2 - a + b = (-a + b)(a + 1)$

Exercise 1 - 7

1. Factorise the following:

(a) $x^2 + 8x + 15$
(b) $x^2 + 10x + 24$
(c) $x^2 + 9x + 8$
(d) $x^2 + 9x + 14$
(e) $x^2 + 15x + 36$
(f) $x^2 + 12x + 36$

2. Write the following expressions in factorised form:

(a) $x^2 - 2x - 15$
(b) $x^2 + 2x - 3$
(c) $x^2 + 2x - 8$
(d) $x^2 + x - 20$
(e) $x^2 - x - 20$
(f) $2x^2 + 22x + 20$

3. Find the factors of the following trinomial expressions:

(a) $3x^2 + 19x + 6$
(b) $6x^2 + 7x + 1$
(c) $12x^2 + 8x + 1$
(d) $8x^2 + 6x + 1$

4. Factorise:

(a) $3x^2 + 17x - 6$
(b) $7x^2 - 6x - 1$
(c) $8x^2 - 6x + 1$
(d) $6x^2 - 15x - 9$

Solutions to Exercise 1 - 7

1. (a) $x^2 + 8x + 15 = (x + 5)(x + 3)$
(b) $x^2 + 10x + 24 = (x + 6)(x + 4)$
(c) $x^2 + 9x + 8 = (x + 8)(x + 1)$
(d) $x^2 + 9x + 14 = (x + 7)(x + 2)$
(e) $x^2 + 15x + 36 = (x + 12)(x + 3)$
(f) $x^2 + 12x + 36 = (x + 6)(x + 6)$

2. (a) $x^2 - 2x - 15 = (x + 5)(x - 3)$
(b) $x^2 + 2x - 3 = (x + 3)(x - 1)$
(c) $x^2 + 2x - 8 = (x + 4)(x - 2)$
(d) $x^2 + x - 20 = (x + 5)(x - 4)$
(e) $x^2 - x - 20 = (x - 5)(x + 4)$
(f) $2x^2 + 22x + 20$
$= 2(x^2 + 11x + 10)$
$= 2(x + 10)(x + 1)$

3.

(a) $3x^2 + 19x + 6 = (3x + 1)(x + 6)$
(b) $6x^2 + 7x + 2 = (6x + 1)(x + 1)$
(c) $12x^2 + 8x + 1 = (6x + 1)(2x + 1)$
(d) $8x^2 + 6x + 1 = (2x + 1)(4x + 1)$

4. (a) $3x^2 + 17x - 6 = (3x + 1)(x + 6)$
   (b) $7x^2 - 6x - 1 = (7x + 1)(x - 1)$
   (c) $8x^2 - 6x + 1 = (4x - 1)(2x - 1)$
   (d) $6x^2 - 15x - 9 = (6x + 3)(x - 3)$

Exercise 1 - 8

Factorise:

1. $x^3 + 8$
2. $27 - m^3$
3. $2x^3 - 2y^3$
4. $3k^3 + 27q^3$
5. $64t^3 - 1$
6. $64x^2 - 1$
7. $125x^3 + 1$
8. $25x^2 + 1$
9. $z - 125z^4$
10. $8m^6 + n^9$
11. $p^{15} - \frac{1}{8}y^{12}$
12. $1 - (x - y)^3$

Solutions to Exercise 1 - 8

1. $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$
2. $27 - m^3 = (3 - m)(9 + 3m + m^2)$
3. $2x^3 - 2y^3$
   $= 2(x^3 - y^3)$
   $= 2(x - y)(x^2 + xy + y^2)$
4. $3k^3 + 27q^3$
   $= 3(k^3 + 27q^3)$
   $= 3(k + 3q)(k^2 - 3kq + 9q^2)$
5. $64t^3 - 1 = (4t - 1)(16t^2 + 4t + 1)$
6. $64x^2 - 1 = (8x - 1)(8x + 1)$
7. $125x^3 + 1 = (5x + 1)(25x^2 - 5x + 1)$
8. No solution (can't be factorised)
9. $z - 125z^4$
   $= z(1 - 125z^3)$
   $= z(1 - 5z)(1 + 5z + 25z^2)$

10. $8m^6 + n^9$
$= (2m^2)^3 + (n^3)^3$
$= (2m^2 + n^3)(4m^4 - 2m^2n^3 + n^6)$

11. $p^{15} - \frac{1}{8}y^{12}$
$= \left(p^5\right)^3 - \left(\frac{1}{2}y^4\right)^3$
$= (p^5 - \frac{1}{2}y^4)(p^{10} + \frac{1}{2}p^5y^4 + \frac{1}{4}y^8$
Or:
$p^{15} - \frac{1}{8}y^{12}$
$= \frac{1}{8}\left(8p^{15} - y^{12}\right) - \frac{1}{8}\left(2p^5 - y^4)(4p^{10} + 2p^5y^4 + y^8\right)$

12. $1 - (x-y)^3$
$= [1 - (x-y)][1 + (1)(x-y) + (x-y)^2]$
$= [1 - x + y][1 + x - y + x^2 - 2xy + y^2]$

Exercise 1 - 9

Simplify (assume all denominators are non-zero):

1. $\frac{3a}{15}$
2. $\frac{2a+10}{4}$
3. $\frac{5a+20}{a+4}$
4. $\frac{a^2-4a}{a-4}$
5. $\frac{3a^2-9a}{2a-6}$
6. $\frac{9a+27}{9a+18}$
7. $\frac{6ab+2a}{2b}$
8. $\frac{16x^2y-8xy}{12x-6}$
9. $\frac{4xyp-8xp}{12xy}$
10. $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$
11. $\frac{a^2-5a}{2a+10} \times \frac{4a}{3a+15}$
12. $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$
13. $\frac{24a-8}{12} \div \frac{9a-3}{6}$
14. $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$
15. $\frac{p^2+pq}{7p} \times \frac{21q}{8p+8q}$
16. $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$
17. $\frac{f^2a-fa^2}{f-a}$
18. $\frac{2}{xy} + \frac{4}{xz} + \frac{3}{yz}$
19. $\frac{5}{t-2} - \frac{1}{t-3}$
20. $\frac{k+2}{k^2+2} - \frac{1}{k+2}$
21. $\frac{t+2}{3q} + \frac{t+1}{2q}$
22. $\frac{3}{p^2-4} + \frac{2}{(p-2)^2}$
23. $\frac{x}{x+y} + \frac{x^2}{y^2-x^2}$
24. $\frac{1}{m+n} + \frac{3mn}{m^3+n^3}$

25. $\frac{h}{h^3 - f^3} - \frac{1}{h^2 + hf + f^2}$

26. $\frac{x^2 - 1}{3} \times \frac{1}{x - 1} - \frac{1}{2}$

27. $\frac{x^2 - 2x + 1}{(x-1)^3} - \frac{x^2 + x + 1}{x^3 - 1}$

28. $\frac{1}{(x-1)^2} - \frac{2x}{x^3 - 1}$

29. $\frac{p^3 + q^3}{p^2} \times \frac{3p - 3q}{p^2 - q^2}$

30. $\frac{1}{a^2 - 4ab + 4b^2} + \frac{a^2 + 2ab + b^2}{a^3 - 8b^3} - \frac{1}{a^2 - 4b^2}$

Solutions to Exercise 1 - 9

1. $\frac{3a}{15} = \frac{a}{5}$

2. $\frac{2a + 10}{4}$

$= \frac{2(a+5)}{4}$

$= \frac{a+5}{2}$

3. $\frac{5a + 20}{a + 4} = 5$

4. $\frac{a^2 - 4a}{a - 4} = a$

5. $\frac{3a^2 - 9a}{2a - 6}$

$= \frac{3a(a-3)}{2(a-3)}$

$= \frac{3a}{2}$

6. $\frac{9a + 27}{9a + 18}$

$= \frac{9(a+3)}{9(a+2)}$

$= \frac{a+3}{a+2}$

7. $\frac{6ab + 2a}{2b}$

$= \frac{2a(3b+1)}{2b}$

$= \frac{a(3b+1)}{b}$

8. $\frac{16x^2y - 8xy}{12x - 6} = \frac{4xy}{3}$

9. $\frac{4xyp - 8xp}{12xy} = \frac{p(y-2)}{3y}$

10. $\frac{3a + 9}{14} \div \frac{7a + 21}{a + 3} = \frac{3(a+3)}{98}$

11. $\frac{a^2 - 5a}{2a + 10} \times \frac{4a}{3a + 15} = \frac{4a^2(a-5)}{6(a+5)^2}$

12. $\frac{3xp + 4p}{8p} \div \frac{12p^2}{3x + 4} = \frac{(3x+4)^2}{96p^2}$

13. $\frac{24a - 8}{12} \div \frac{9a - 3}{6} = \frac{4}{3}$

14. $\frac{a^2 + 2a}{5} \div \frac{2a + 4}{20} = 2a$

15. $\frac{p^2 + pq}{7p} \times \frac{21q}{8p + 8q} = \frac{3q}{8}$

16. $\frac{5ab - 15b}{4a - 12} \div \frac{6b^2}{a + b} = \frac{5(a+b)}{24b}$

17. $\frac{f^2a - fa^2}{f - a}$

$= \frac{af(f-a)}{f-a}$

$= af$

18. $\frac{2}{xy} + \frac{4}{xz} + \frac{3}{yz}$

$= \frac{2z}{xyz} + \frac{4y}{xyz} + \frac{3x}{xyz}$

$= \frac{2z + 4y + 3x}{xyz}$

19. $\frac{5}{t-2} - \frac{1}{t-3}$

$= \frac{5(t-3)}{(t-2)(t-3)} - \frac{t-2}{(t-2)(t-3)}$

$= \frac{5(t-2)(t-3)}{(t-2)(t-3)}$

$= 5$

20. $\frac{k+2}{k^2+2} - \frac{1}{k+2}$

$= \frac{(k+2)(k+2)-(k^2+2)}{(k^2+2)(k+2)}$

$= \frac{k^2+4k+4-k^2-2}{k^3+2k^2+2k+4}$

$= \frac{2(k-1)}{(k^2+2)(k+2)}$

21. $\frac{t+2}{3q} + \frac{t+1}{2q}$

$= \frac{2(t+2)+3(t+1)}{6q}$

$= \frac{5t+7}{6q}$

22. $\frac{3}{p^2-4} + \frac{2}{(p-2)^2}$

$= \frac{3(p-2)+2(p+2)}{(p-2)(p+2)(p-2)}$

$= \frac{(p-2)+2(p+2)}{(p+2)(p^2-4)}$

$= \frac{(5p-2)}{(p-2)^2(p+2)}$

23. $\frac{x}{x+y} + \frac{x^2}{y^2-x^2}$

$= \frac{x(x-y)-x^2}{(x+y)(x-y)}$

$= \frac{-xy}{x^2-y^2}$

24. $\frac{1}{m+n} + \frac{3mn}{m^3+n^3}$

$= \frac{1}{m+n} + \frac{3mn}{(m+n)(m^2-mn+n^2)}$

$= \frac{m^2-mn+n^2+3mn}{m^3-m^3}$

$= \frac{m^2+2mn+n^2}{m^3-n^3}$

$= \frac{m+1}{m^2-mn+n^2}$

25. $\frac{h}{h^3-f^3} - \frac{1}{h^2+hf+f^2}$

$= \frac{h}{(h-f)(h^2+hf+f^2)} - \frac{1}{h^2+hf+f^2}$

$= \frac{h-(h-f)}{h^3-f^3}$

$= \frac{f}{h^3 - f^3}$

26. $\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2}$
$= \frac{2x-1}{6}$

27. $\frac{x^2-2x+1}{(x-1)^3} - \frac{x^2+x+1}{x^3-1}$
$= \frac{(x-1)^2}{(x-1)^3} - \frac{x^2+x+1}{x^3-1}$
$= \frac{1}{(x-1)} - \frac{x^2+x+1}{(x-1)(x^2+x+1)}$
$= \frac{1}{(x-1)} - \frac{1}{(x-1)}$
$= 0$

28. $\frac{1}{(x-1)^2} - \frac{2x}{x^3-1}$
$= \frac{1}{(x-1)^2} - \frac{2x}{(x-1)(x^2+x+1)}$
$= \frac{x^2+x+1-2x(x-1)}{(x-1)^2(x^2+x+1)}$
$= \frac{x^2+x+1-2x^2-2x}{(x-1)^2(x^2+x+1)}$
$= \frac{-x^2+3x+1}{(x-1)^2(x^2+x+1)}$
$= -\frac{x^2-3x-1}{(x-1)^2(x^2+x+1)}$

29. $\frac{p^3+q^3}{p^2} \times \frac{3p-3q}{p^2-q^2}$
$= \frac{(p+q)(p^2-pq+q^2)}{p^2} \times \frac{3(p-q)}{(p-q)(p+q)}$
$= \frac{3(p^2-pq+q^2)}{p^2}$

30. $\frac{1}{a^2-4ab+4b^2} + \frac{a^2+2ab+b^2}{a^3-8b^3} - \frac{1}{a^2-4b^2}$
$= \frac{1}{(a-2b)(a-2b)} + \frac{a^2+2ab+4b^2}{(a-2b)(a^2+2ab+4b^2} - \frac{1}{(a-2b)(a+2b)}$
$= \frac{(a+2b)+(a-2b)(a+2b)-(a-2b)}{(a-2b^2)(a+2b)}$
$= \frac{a+2b+a^2-4b^2-a+2b}{(a-2b^2)(a+2b)} = \frac{a^2+4b-4b^2}{(a-2b^2)(a+2b)}$

End of Chapter Exercises

1. If a is an integer, b is an integer and c is irrational, which of the following are rational numbers?

(a) $\dfrac{-b}{a}$

(b) $c \div c$

(c) $\dfrac{a}{c}$

(d) $\dfrac{1}{c}$

2. Write each decimal as a simple fraction.

(a) $0{,}12$

(b) $0{,}006$

(c) $1{,}59$

(d) $12{,}27\dot{7}$

3. Show that the decimal $3{,}21\dot{1}\dot{8}$ is a rational number.

4. Express $0{,}7\dot{8}$ as a fraction $\dfrac{a}{b}$ where $a,b \in \mathbb{Z}$ (show all working).

5. Write the following rational numbers to 2 decimal places.

(a) $\dfrac{1}{2}$

(b) 1

(c) $0{,}11111\overline{1}$

(d) $0{,}99999\overline{1}$

6. Round off the following irrational numbers to 3 decimal places.

(a) $3{,}141592654\ldots$

(b) $1{,}618033989\ldots$

(c) $1{,}41421356\ldots$

(d) $2{,}71828182845904523536\ldots$

7. Use your calculator and write the following irrational numbers to 3 decimal places.

(a) $\sqrt{2}$

(b) $\sqrt{3}$

(c) $\sqrt{5}$

(d) $\sqrt{6}$

8. Use your calculator (where necessary) and write the following numbers to 5 decimal places. State whether the numbers are irrational or rational.

(a) $\sqrt{8}$

(b) $\sqrt{768}$

(c) $\sqrt{0{,}49}$

(d) $\sqrt{0{,}0016}$

(e) $\sqrt{0{,}25}$

(f) $\sqrt{36}$

(g) $\sqrt{1960}$

(h) $\sqrt{0{,}0036}$

(i) $-8\sqrt{0{,}04}$

(j) $5\sqrt{80}$

9. Write the following irrational numbers to 3 decimal places and then write each one as a rational number to get an approximation to the irrational number.

(a) $3{,}141592654\ldots$

(b) $1{,}618033989\ldots$

(c) $1{,}41421356\ldots$

(d) $2{,}71828182845904523536\ldots$

10. Determine between which two consecutive integers the following irrational numbers lie, without using a calculator.

(a) $\sqrt{5}$

(b) $\sqrt{10}$

(c) $\sqrt{20}$

(d) $\sqrt{30}$

(e) $\sqrt[3]{5}$

(f) $\sqrt[3]{10}$

(g) $\sqrt[3]{20}$

(h) $\sqrt[3]{30}$

11. Find two consecutive integers such that $\sqrt{7}$ lies between them.

12. Find two consecutive integers such that $\sqrt{15}$ lies between them.

13. Factorise:

    (a) $a^2 - 9$
    (b) $m^2 - 36$
    (c) $9b^2 - 81$
    (d) $16b^6 - 25a^2$
    (e) $m^2 - \frac{1}{9}$
    (f) $5 - 5a^2b^6$
    (g) $16ba^4 - 81b$
    (h) $a^2 - 10a + 25$
    (i) $16b^2 + 56b + 49$
    (j) $2a^2 - 12ab + 18b^2$
    (k) $-4b^2 - 144b^8 + 48b^5$
    (l) $(16 - x^4)$
    (m) $7x^2 - 14x + 7xy - 14y$
    (n) $y^2 - 7y - 30$
    (o) $1 - x - x^2 + x^3$
    (p) $-3(1 - p^2) + p + 1$
    (q) $x - x^3 + y - y^3$
    (r) $x^2 - 2x + 1 - y^4$
    (s) $4b(x^3 - 1) + x(1 - x^3)$
    (t) $3p^3 - \frac{1}{9}$
    (u) $8x^6 - 125y^9$
    (v) $(2 + p)^3 - 8(p + 1)^3$

14. Simplify the following:

    (a) $(a - 2)^2 - a(a + 4)$
    (b) $(5a - 4b)(25a^2 + 20ab + 16b^2)$
    (c) $(2m - 3)(4m^2 + 9)(2m + 3)$
    (d) $(a + 2b - c)(a + 2b + c)$
    (e) $\dfrac{p^2 - q^2}{p} \div \dfrac{p + q}{p^2 - pq}$
    (f) $\dfrac{2}{x} + \dfrac{x}{2} - \dfrac{2x}{3}$
    (g) $\dfrac{1}{a + 7} - \dfrac{a + 7}{a^2 - 49}$
    (h) $\dfrac{x + 2}{2x^3} + 16$
    (i) $\dfrac{1 - 2a}{4a^2 - 1} - \dfrac{a - 1}{2a^2 - 3a + 1} - \dfrac{1}{1 - a}$
    (j) $\dfrac{x^2 + 2x}{x^2 + x + 6} \times \dfrac{x^2 + 2x + 1}{x^2 + 3x + 2}$

15. Show that $(2x - 1)^2 - (x - 3)^2$ can be simplified to $(x + 2)(3x - 4)$.

16. What must be added to $x^2 - x + 4$ to make it equal to $(x + 2)^2$?

17. Evaluate $\dfrac{x^3 + 1}{x^2 - x + 1}$ if $x = 7{,}85$ without using a calculator. Show your work.

18. With what expression must $(a - 2b)$ be multiplied to get a product of $a^3 - 8b^3$?

19. With what expression must $27x^3 + 1$ be divided to get a quotient of $3x + 1$?

Solutions to End of Chapter Exercises

1. (a) Rational
   (b) Irrational
   (c) Irrational
   (d) Irrational

2. (a) $0{,}12$
   $= \frac{1}{10} + \frac{2}{100}$

$= \frac{12}{100}$
$= \frac{6}{10}$
$= \frac{3}{5}$

(b) $0{,}006$
$= \frac{6}{1000}$
$= \frac{3}{500}$

(c) $1{,}59$
$= 1 + \frac{5}{10} + \frac{9}{100}$
$= 1\frac{59}{100}$

(d) $x = 12{,}2\dot{7}$
$10x = 122{,}\dot{7}$
$100x = 1\ 227{,}\dot{7}$
$\therefore 100x - 10x = 90x = 1105$
$\therefore x = \frac{1105}{90} = \frac{221}{18}$

3. $x = 3{,}21\overline{18}$
$10\ 000x = 32\ 118{,}\overline{18}$
$\therefore 10\ 000x - x = 9\ 999x = 32115$
$\therefore x = \frac{32\ 115}{9\ 999}$
This is a rational number because both the numerator and denominator are integers.

4. $x = 0{,}\overline{78}$
$100x = 78{,}\overline{78}$
$\therefore 100x - x = 99x = \frac{78}{99}$

5. **(a)** To write to two decimal places we must convert to decimal: $\frac{1}{2} = 0{,}5$
(b) To write to two decimal places just add a comma and two 0's: $1{,}00$
(c) We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer. In this case there is a 1 after the cut off point so we do not round up. The final answer is: $0{,}11111\overline{1} \sim 0{,}11$
(d) Repeat the steps in c) but this time we round up: $0{,}99999\overline{1} \sim 1{,}00$

6. We mark where the cut off point is, determine if it has to be rounded up or not and then write the answer.
(a) $3{,}142$ (round up as there is a 5 after the cut off point)
(b) $1{,}618$ (no rounding as there is a 0 after the cut off point)
(c) $1{,}414$ (no rounding as there is a 2 after the cut off point)
(d) $2{,}718$ (round up as there is a 2 after the cut off point)

7. **(a)** $1{,}414$
(b) $1{,}732$
(c) $2{,}236$
(d) $2{,}449$

8. **(a)** $2{,}82843$
(b) $27{,}71281$ - irrational
(c) $10{,}00000$ - rational
(d) $0{,}70000$ - rational
(e) $0{,}04000$ - rational
(f) $0{,}500000$ - rational
(g) $6{,}00000$ - rational
(h) $44{,}27189$ - irrational
(i) $0{,}06000$ - rational
(j) $-8(0{,}2) = -4{,}00000$ - rational
(k) $44{,}72136$ - irrational

9. (a) $3\frac{142}{1\ 000} = \frac{1\ 571}{500}$
(b) $1\frac{618}{1000} = \frac{809}{500}$
(c) $1\frac{414}{1000}\frac{707}{500}$
(d) $2\frac{718}{1000} = \frac{1\ 359}{500}$

10. (a) 2 and 3
(b) 3 and 4
(c) 4 and 5
(d) 5 and 6
(e) 1 and 2
(f) 2 and 3
(g) 2 and 3
(h) 3 and 4

11. 2 and 3

12. 3 and 4

13. (a) $a^2 - 9 = (a-3)(a+3)$

(b) $m^2 - 36 = (m+6)(m-6)$

(c) $9b^2 - 81 = (3b-9)(3b+9)$

(d) $16b^6 - 25a^2 = (4b+5a)(4b-5a)$

(e) $m^2 - \frac{1}{9} = (m+\frac{1}{3})(m-\frac{1}{3})$

(f) $5 - 5a^2b^6$
$= 5(1-a^2b^6)$
$= 5(1-ab^3)(1+ab^3)$

(g) $16ba^4 - 81b$
$= b(4a^2+9)(4a^2-9)$
$= b(4a^2+9)(2a+3)(2a-3)$

(h) $a^2 - 10a + 25 = (a-5)(a-5)$

(i) $16b^2 + 56b + 49 = (4b+7)(4b+7)$

(j) $2a^2 - 12ab + 18b^2 = (2a-6b)(a-3b)$

(k) $-4b^2 - 144b^8 + 48b^5$
$= -4b^2(6b^3-1)(6b^3-1)$
$= -4b^2(6b^3-1)^2$

(l) $(16-x^4)$
$= (4-x^2)(4+x^2)$
$= (2-x)(2+x)(4+x^2)$

(m) $7x^2 - 14x + 7xy - 14y$
$= 7x(x-2) + 7y(x-2)$
$= (x-2)(7x+7y)$
$= 7(x-2)(x+y)$

(n) $y^2 - 7y - 30 = (y-10)(y+3)$

(o) $1 - x - x^2 + x^3$
$= (1-x) - x^2(1-x)$
$= (1-x)(1-x^2)$
$= (1-x)^2(1+x)$

(p) $-3(1-p^2) + p + 1$
$= -3(1-p)(1+p) + (p+1)$
$= (p+1)[-3(1-p)+1]$

$= (p-1)(-3p-2)$

(q) $x - x^3 + y - y^3$
$= (x+y) - (x^3+y^3)$
$= (x+y) - (x+y)(x^2 - xy + y^2)$
$= (x+y)(1 - x^2 + xy - y^2)$

(r) $x^2 - 2x + 1 - y^4$
$= x(x-2) + (1-y^2)(1+y^2)$
$= x(x-2) + (1+y)(1-y)(1+y^2)$

(s) $4b(x^3-1) + x(1-x^3)$
$= (x^3-1)(4b-x)$
$= (x-1)(x^2+x+1)(4b-x)$

(t) $3p^3 - \frac{1}{9} = 3(p - \frac{1}{3})(p^2 + \frac{p}{3} + \frac{1}{9})$

(u) $8x^6 - 125y^9 = (2x^2 - 5y^3)(4x^4 + 10x^2y^3 + 25y^6)$

(v) $(2+p)^3 - 8(p+1)^3$
$= [(p+2) - 2(p+1)][(p+2)^2 + 2(p+2)(p+1) + 4(p+1)^2]$
$= [p + 2 - 2p - 2][p^2 + 4p + 4 + 2p^2 + 6p + 4 + 4p^2 + 8p + 4]$
$= (-p)(12 + 18p + 7p^2)$

14. **(a)** $(a-2)^2 - a(a+4)$
$= a^2 - 4a + 4 - a^2 - 4a$
$= -8a + 4$

(b) $(5a - 4b)(25a^2 + 20ab + 16b^2)$
$= 125a^3 - 64b^3$

(c) $(2m-3)(4m^2+9)(2m+3)$
$= (4m^2-9)(4m^2+9)$
$= 16m^4 - 81$

(d) $(a + 2b - c)(a + 2b + c)$
$= (a+2b)^2 - c^2$
$= a^2 + 4ab + 4b^2 - c^2$

(e) $\dfrac{p^2-q^2}{p} \div \dfrac{p+q}{p^2-pq}$
$= \dfrac{(p-q)(p+q)}{p} \times \dfrac{p(p-q)}{p+q}$
$= (p-q)^2$
$= p^2 - 2pq + q^2$

(f) $\dfrac{2}{x} + \dfrac{x}{2} - \dfrac{2x}{3}$
$= \dfrac{12 + 3x^2 - 4x^2}{6x}$
$= \dfrac{12 - x^2}{6x}$

(g) $\dfrac{1}{a+7} - \dfrac{a+7}{a^2-49}$
$= \dfrac{1}{a+7} - \dfrac{a+7}{(a+7)(a-7)}$

$= \frac{-14}{(a+7)(a-7)}$

(h) $\frac{x+2}{2x^3} + 16$

$= \frac{(x+2)+16(2x^3)}{2x^3}$

$= \frac{32x^3+x+2}{2x^3}$

(i) $\frac{1-2a}{4a^2-1} - \frac{a-1}{2a^2-3a+1} - \frac{1}{1-a}$

$= \frac{1-2a}{(2a-1)(2a+1)} - \frac{a-1}{(2a-1)(2a+1)} + \frac{1}{a-1}$

$= -\frac{(2a-1)}{(2a-1)(2a+1)} - \frac{1}{2a-1} + \frac{1}{a-1}$

$= \frac{4a-1}{(2a+1)(2a-1)(a-1)}$

(j) $\frac{x^2+2x}{x^2+x+6} \times \frac{x^2+2x+1}{x^2+3x+2}$

$= \frac{x(x+2)}{x^2+x+6} \times \frac{(x+1)(x+1)}{(x+2)(x+1)}$

$= \frac{x(x+1)}{x^2+x+6}$

15. $(2x-1)(2x-1)-(x-3)(x-3)$
$= 4x^2-2x-2x+1-(x^2-3x-3x-9)$
$= 3x^2+2x-8$
$= (3x-4)(x+2)$

16. Suppose A must be added to the expression to get the desired result.
$\therefore (x^2-x+4)+A = (x+2)^2$
$\therefore A = (x+2)(x+2)-(x^2-x+4)$
$= x^2+2x+2x+4-x^2+x-4$
$= 5x$
Therefore $5x$ must be added.

17. First simplify the expression: $\frac{(x+1)(x^2-x+1)}{x^2-x+1} = x+1$

Now substitute the value of x: $7{,}85+1 = 8{,}85$

18. $(a-2b)(a^2+2ab+4b^2) = a^3-8b^3$
so, the expression is $a^2+2ab+4b^2$.

19. $27x^3+1 = (3x+1)(9x^2-3x+1)$
$\frac{(3x+1)(9x^2-3x+1)}{9x^2-3x+1} = 3x+1$
so, the expression is $9x^2-3x+1$.

Equations and inequalities 2

Exercise 2 - 1

Solve the following equations (assume all denominators are non-zero):

1. $2y - 3 = 7$
2. $-3y = 0$
3. $16y + 4 = -10$
4. $12y + 0 = 144$
5. $7 + 5y = 62$
6. $55 = 5x + \frac{3}{4}$
7. $5x = 2x + 45$
8. $23x - 12 = 6 + 3x$
9. $12 - 6x + 34x = 2x - 24 - 64$
10. $6x + 3x = 4 - 5(2x - 3)$
11. $18 - 2p = p + 9$
12. $\frac{4}{p} = \frac{16}{24}$
13. $-(-16 - p) = 13p - 1$
14. $3f - 10 = 10$
15. $3f + 16 = 4f - 10$
16. $10f + 5 = -2f - 3f + 80$
17. $8(f - 4) = 5(f - 4)$
18. $6 = 6(f + 7) + 5f$
19. $(a - 1)^2 - 2a = (a + 3)(a - 2) - 3$
20. $-7x = x + 8(1 - x)$
21. $5 - \frac{7}{b} = \frac{2(b + 4)}{b}$
22. $\frac{x + 2}{4} - \frac{x - 6}{3} = \frac{1}{2}$
23. $3 - \frac{y - 2}{4} = 4$
24. $\frac{a + 1}{a + 2} = \frac{a - 3}{a + 1}$
25. $(x - 3)(x + 2) = x(x - 4)$
26. $1{,}5x + 3{,}125 = 1{,}25x$
27. $\frac{1}{3}P + \frac{1}{2}P - 10 = 0$
28. $1\frac{1}{4}(x - 1) - 1\frac{1}{2}(3x + 2) = 0$
29. $\frac{5}{2a} + \frac{1}{6a} - \frac{3}{a} = 2$
30. $\frac{3}{2x^2} + \frac{4}{3x} - \frac{5}{6x} = 0$

Solutions to Exercise 2 - 1

1. $2y - 3 = 7$
   $2y = 10$
   $y = 5$

2. $-3y = 0$
   $y = \frac{0}{3}$
   $y = 0$

3. $16y + 4 = -10$
   $16y = -14$
   $y = -\frac{14}{16}$
   $= -\frac{7}{8}$

4. $12y + 0 = 144$
   $y = \frac{144}{12}$
   $y = 12$

5. $7 + 5y = 62$
   $5y = 62 - 7$
   $5y = 11$
   $y = 11$

6. $55 = 5x + \frac{3}{4}$
   $220 = 20x + 3$
   $20x = 117$
   $x = \frac{117}{20}$

7. $5x = 2x + 45$
   $3x = 45$
   $x = 15$

8. $23x - 12 = 6 + 3x$
   $20x = 18$
   $x = \frac{18}{20}$
   $x = \frac{9}{10}$

9. $12 - 6x + 34x = 2x - 24 - 64$
   $-6x + 34x - 2x = -24 - 64 - 12$
   $26x = 100$
   $x = \frac{100}{26}$

10. $6x + 3x = 4 - 5(2x - 3)$
    $6x + 3x = 4 - 10x + 15$
    $19x = 19$
    $x = 1$

11. $18 - 2p = p + 9$
    $9 = 3p$
    $p = 3$

12. $\frac{4}{p} = \frac{16}{24}$
    $96 = 16p$
    $p = 6$

13. $-(-16 - p) = 13p - 1$
    $16 + p = 13p - 1$
    $17 = 12p$
    $p = \frac{17}{12}$

14. $3f - 10 = 10$
    $3f = 20$
    $f = \frac{20}{3}$

15. $3f + 16 = 4f - 10$
    $-f = -10 - 16$
    $f = 26$

16. $10f + 5 = -2f - 3f + 80$
    $10f + 2f + 3f = 75$
    $15f = 75$
    $f = 5$

17. $8(f - 4) = 5(f - 4)$
    $8f - 32 = 5f - 20$
    $3f = 12$
    $f = 4$

18. $6 = 6(f + 7) + 5f$
    $6 = 6f + 42 + 5f$
    $-36 = 11f$
    $f = -\frac{36}{11}$

19. $(a - 1)^2 - 2a = (a + 3)(a - 2) - 3$
    $a^2 - 2a + 1 - 2a = a^2 - 2a + 3a - 6 - 3$
    $-4a + 1 = a - 9$
    $5a = 10$
    $a = 2$

20. $-7x = x + 8(1 - x)$
    $-7x = -7x + 8$
    $0 = 8$
    No solution

21. $5 - \dfrac{7}{b} = \dfrac{2(b+4)}{b}$
    $5b - 7 = 2(b + 4)$
    $5b - 7 = 2b + 8$
    $3b = 15$
    $b = 5$

22. $\dfrac{x+2}{4} - \dfrac{x-6}{3} = \dfrac{1}{2}$
    $\dfrac{3(x=2)}{12} - \dfrac{4(x-6)}{12} = \frac{6}{12}$
    $3(x + 2) - 4(x - 6) = 6$
    $3x + 6 - 4x + 24 = 6$
    $-x = -24$
    $x = 24$

23. $3 - \dfrac{y-2}{4} = 4$
    $12 - y - 2 = 16$
    $-4 = y + 2$

$y = -6$

24. $\dfrac{a+1}{a+2} = \dfrac{a-3}{a+1}$
$\dfrac{a+1}{a+2} - \dfrac{a-3}{a+1} = 0$
$(a+1)^2 - (a-3)(a+2) = 0$
$a^2 + 2a + 1 - (a^2 - a - 6) = 0$
$a^2 + 2a + 1 - a^2 + a + 6 = 0$
$3a + 7 = 0$
$a = -\frac{7}{3}$

25. $(x-3)(x+2) = x(x-4)$
$x^2 - x - 6 = x^2 - 4x$
$-x + 4x = 6$
$3x = 6$
$x = 2$

26. $1{,}5x + 3{,}125 = 1{,}25x$
$1{,}5x - 1{,}25x = -3{,}125$
$0{,}25x = -3{,}125$
$x = -12{,}5$

27. $\frac{1}{3}P + \frac{1}{2}P - 10 = 0$
$2p + 3p = 10$
$5p = 10$
$p = 2$

28. $1\frac{1}{4}(x-1) - 1\frac{1}{2}(3x+2) = 0$
$\frac{5}{4}x - \frac{5}{4} - \frac{9}{2}x - 3 = 0$
$\frac{5x-8x}{4} = \frac{5+12}{4}$
$-13x = 17$
$x = -\frac{17}{13}$

29. $\frac{5}{2a} + \frac{1}{6a} - \frac{3}{a} = 2$
$15 + 1 - 18 = 12a$
$-2 = 12a$
$a = -\frac{1}{6}$

30. $\frac{3}{2x^2} + \frac{4}{3x} - \frac{5}{6x} = 0$
$9 + 8x - 5x = 0$
$9 + 3x$
$3x = -9$
$x = -3$

Exercise 2 - 2

Solve the following equations:

1. $9x^2 - 6x - 8 = 0$

2. $5x^2 - 21x - 54 = 0$

3. $4y^2 - 9 = 0$

4. $4x^2 - 16x - 9 = 0$

5. $4x^2 - 12x = -9$

6. $20m + 25m^2 = 0$

7. $2x^2 - 5x - 12 = 0$

8. $-75x^2 + 290x = 240$

9. $2x = \frac{1}{3}x^2 - 3x + 14\frac{2}{3}$

10. $x^2 - 4x = -4$

11. $-x^2 + 4x - 6 = 4x^2 - 14x + 3$

12. $t^2 = 3t$

13. $x^2 - 10x = -25$

14. $x^2 = 18$

15. $p^2 - 6p = 7$

16. $4x^2 - 17x - 77 = 0$

17. $14x^2 + 5x = 6$

18. $2x^2 - 2x = 12$

19. $\frac{a+1}{3a-4} + \frac{9}{2a+5} + \frac{2a+3}{2a+5} = 0$

20. $\frac{3}{9a^2-3a+1} - \frac{3a+4}{27a^3+1} = \frac{1}{9a^2-1}$

Solutions to Exercise 2 - 2

1. $(3x+2)(3x-4)=0$
   $\therefore x=-\frac{2}{3}$ or $x=\frac{4}{3}$

2. $(5x-9)(x+6)=0$
   $\therefore x=\frac{9}{5}$ or $x=-6$

3. $(2y+3)(2y-3)=0$
   $\therefore y=\frac{-3}{2}$ or $y=\frac{3}{2}$

4. $x(2x+1)(2x-9)$
   $\therefore x=-\frac{1}{2}$ or $x=\frac{9}{2}$

5. $(4x)(x-3)=-9$
   $4x^2-12x+9=0$
   $(2x-3)(2x-3)=0$
   $\therefore x=\frac{3}{2}$

6. $20m+25m^2=0$
   $5m(4+5m)=0$
   $\therefore m=0$ or $m=\frac{-4}{5}$

7. $2x^2-5x-12=0$
   $(2x+3)(x-4)=0$
   $\therefore x=4$ or $\frac{-3}{2}$

8. $-75x^2+290x=240$
   $-15x^2+58x-48=0$
   $(5x-6)(3x-8)$
   $\therefore x=\frac{6}{5}$ or $x=\frac{8}{3}$

9. $2x=\frac{1}{3}x^2-3x+14\frac{2}{3}$
   $2x=\frac{1}{3}x^2-3x+\frac{44}{3}$
   $6x=x^2-9x+44$
   $x^2-15x+44=0$
   $(x-4)(x-11)=0$
   $\therefore x=4$ or $x=11$

10. $x^2-4x=-4$
    $(x-2)(x-2)=0$
    $\therefore x=2$

11. $-x^2+4x-6=4x^2-14x+3$
    $5x^2-18x+9=0$
    $(5x-3(x-3)=0$
    $\therefore x=\frac{3}{5}$ or $x=3$

12. $t^2 = 3t$
$t(t-3) = 0$
$\therefore t = 0$ or $t = 3$

13. $x^2 - 10x = -25$
$x^2 - 10x + 25 = 0$
$(x-5)(x-5) = 0$
$\therefore x = 5$

14. $x^2 = 18$
$\therefore x = \sqrt{18}$ or $x = -\sqrt{18}$

15. $p^2 - 6p = 7$
$p^2 - 6p - 7 = 0$
$(p+1)(p-7) = 0$
$\therefore p = -1$ or $p = 7$

16. $4x^2 - 17x - 77 = 0$
$(4x+11)(x-7) = 0$
$\therefore x = 7$ or $x = -\frac{11}{4}$

17. $14x^2 + 5x = 6$
$14x^2 + 5x - 6 = 0$
$(7x-3)(2x+2) = 0$
$\therefore x = \frac{3}{7}$ or $x = -\frac{1}{2}$

18. $2x^2 - 2x = 12$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$\therefore x = -2$ or $x = 3$

19. $\dfrac{a+1}{3a-4} + \dfrac{9}{2a+5} + \dfrac{2a+3}{2a+5} = 0$
$\dfrac{(a+1)(2a+5) + 9(3a-4) + (2a+3)(3a-4)}{(3a-4)(2a+5)} = 0$
$2a^2 + 7a + 5 + 27a - 36 + 6a^2 + a - 12 = 0$
$8a^2 + 35a - 43 = 0$
$(8a+43)(a-1) = 0$
$a = 1$ or $a = \frac{-43}{8} = -5\frac{3}{8}$

20. $\dfrac{3}{9a^2-3a+1} - \dfrac{3a+4}{27a^3+1} = \dfrac{1}{9a^2-1}$
$\dfrac{3}{9a^2-3a+1} - \dfrac{3a+4}{(3a+1)(9a^2-3a+1)} = \dfrac{1}{(3a-1)(3a+1)}$
$\dfrac{3(9a^2-1) - (3a-1)(3a+4)}{(9a^2-3a+1)(3a+1)(3a-1)} = \dfrac{9a^2-3a+1}{(9a^2-3a+1)(3a+1)(3a-1)}$
$27a^2 - 3 - 9a^2 - 9a + 4 = 9a^2 - 3a + 1$
$9a^2 - 6a = 0$
$3a(3a-2) = 0$
$a = 0$ or $a = \frac{2}{3}$

Exercise 2 - 3

1. Solve for x and y:
   (a) $3x - 14y = 0$ and $x - 4y + 1 = 0$
   (b) $x + y = 8$ and $3x + 2y = 21$
   (c) $y = 2x + 1$ and $x + 2y + 3 = 0$
   (d) $\frac{a}{2} + b = 4$ and $\frac{a}{4} - \frac{b}{4} = 1$
   (e) $\frac{1}{x} + \frac{1}{y} = 3$ and $\frac{1}{x} - \frac{1}{y} = 11$
2. Solve graphically and check your answer algebraically:
   (a) $x + 2y = 1$ and $\frac{x}{3} + \frac{y}{2} = 1$
   (b) $5 = x + y$ and $x = y - 2$
   (c) $3x - 2y = 0$ and $x - 4y + 1 = 0$
   (d) $\frac{x}{4} = \frac{y}{2} - 1$ and $\frac{y}{4} + \frac{x}{2} = 1$
   (e) $2x + y = 5$ and $3x - 2y = 4$

Solutions to Exercise 2 - 3

1. Solve algebraically:
   (a) $3x - 14y = 0$
   $\therefore 3x = 14y$
   $\therefore x = \frac{14}{3}y$
   Substitute value of x into second equation:
   $x - 4y + 1 = 0$
   $\therefore \frac{14}{3}y - 14y + 1 = 0$
   $14y - 12y + 3 = 0$
   $2y = -3$
   $y = -\frac{3}{2}$
   Substitute value of y back into first equation:
   $\therefore x = \frac{14(-\frac{3}{2})}{3} = -7$
   $y = -\frac{3}{2}$ and $x = -7$.
   (b) $x + y = 8$
   $\therefore x = 8 - y$
   Substitute value of x into second equation:
   $3x + 2y = 21$
   $\therefore 3(8 - y) + 2y = 21$
   $24 - 3y + 2y = 21$
   $-y = -3$

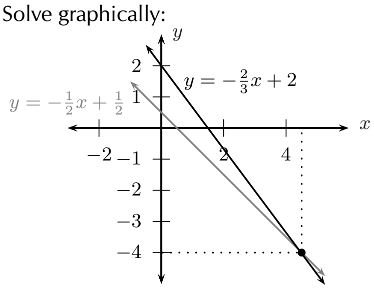
$y = 3$
Substitute value of y back into first equation:
$\therefore x = 8 - 3 = 5$
$y = 3$ and $x = 5$

(c) $y = 2x + 1$
Substitute value of y into second equation:
$x + 2y + 3 = 0$
$\therefore x + 2(2x + 1) + 3 = 0$
$x + 4x + 2 + 3 = 0$
$5x = -5$
$\therefore x = -1$
Substitute value of x back into first equation:
$\therefore y = 2(-1) + 1 = -1$
$y = -1$ and $x = -1$

(d) $\frac{a}{2} + b = 4$
$a + 2b = 8$
$a - b = 4$
$3b = 4$
$b = \frac{4}{3}$
Substitute $b = \frac{4}{3}$ into the first equation:
$\frac{a}{2} + \frac{4}{3} = 4$
$\frac{a}{2} = 4 - \frac{4}{3}$
$\frac{a}{2} = \frac{8}{3}$
$a = \frac{16}{3}$
$a = 5\frac{1}{3}$ and $b = 1\frac{1}{3}$

(e) $\frac{1}{x} + \frac{1}{y} = 3$
$y + x = 3xy$
$y - x = 11xy$
$2y = 14xy$
$\frac{2y}{14y} = x$
$x = \frac{1}{7}$
Substitute $x = \frac{1}{7}$ into the first equation:
$7 + \frac{1}{y} = 3$
$\frac{1}{y} = -4$
$y = -\frac{1}{4}$
$x = \frac{1}{7}$ and $y = -\frac{1}{4}$

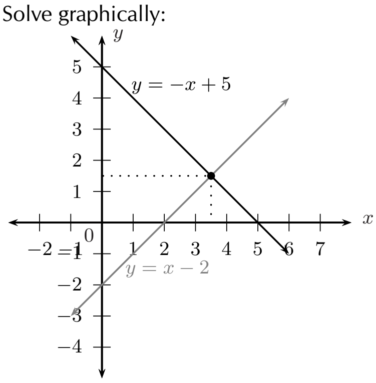
2. **(a)** Solve graphically:



To check algebraically:

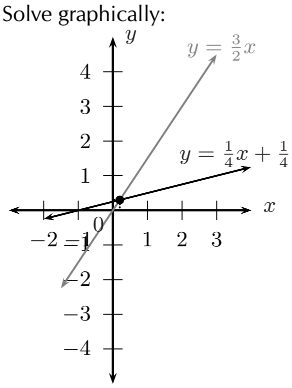
$x + 2y = 1$
$\therefore x = -2y + 1$
Substitute value of x into the second equation:
$\therefore \frac{-2y+1}{3} + \frac{y}{2} = 1$
$2(-2y + 1) + 3y = 6$
$-4y + 2 + 3y = 6$
$\therefore y = -4$
Substitute value of y back into the first equation:
$x + 2(-4) = 1$
$x - 8 = 1$
$\therefore x = 9$

(b) Solve graphically:



To check algebraically:
$5 = x + y$
$\therefore y = 5 - x$
Substitute value of y into second equation:
$x = 5 - x - 2$
$2x = 3$
$\therefore x = \frac{3}{2}$
Substitute value of x back into first equation:
$5 = \frac{3}{2} + y$
$y = \frac{10}{2} - \frac{3}{2} = \frac{7}{2}$

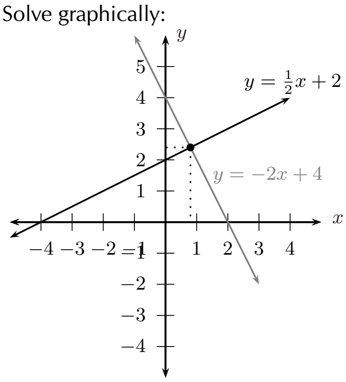
(c) Solve graphically:



To check algebraically:

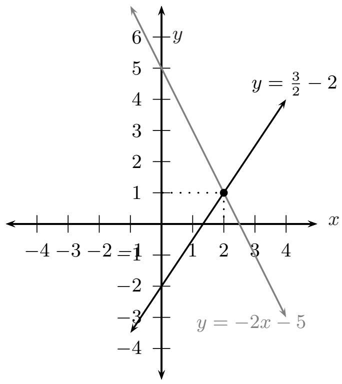
$3x - 2y = 0$
$\therefore y = \frac{3}{2}x$
Substitute value of y into second equation:
$\therefore x - 4\left(\frac{3}{2}x\right) + 1 = 0$
$x - 6x + 1 = 0$
$5x = 1$
$x = \frac{1}{5}$
Substitute value of x back into first equation:
$3\frac{1}{5} - 2y = 0$
$3 - 10y = 0$
$10y = 3$
$y = \frac{3}{10}$

(d) Solve graphically:



To check algebraically:
$\frac{x}{4} = \frac{y}{2} - 1$
$x = 2y - 4$
$y = \frac{1}{2}x + 2$
$\frac{y}{4} + \frac{x}{2} = 1$
$y + 2x = 4$
$y = -2x + 4$
Substitute $y = \frac{1}{2}x + 2$ into $y = -2x + 4$
$\frac{1}{2}x + 2 = -2x + 4$
$2\frac{1}{2}x = 2$
$x = \frac{4}{5}$
Substitute into $y = -2x + 4$
$y = -2\frac{4}{5} + 4$
$= -\frac{8}{5} + 4$
$= 2\frac{2}{5}$

(e) Solve graphically:



$y = -2x + 5$

$y = \frac{3}{2}x - 2$

$0 = -\frac{7}{2}x + 7$

$\frac{7}{2}x = 7$

$x = 2$

Substitute $x = 2$ into the first equation:

$2(2) + y = 5$

$y = 1$

Exercise 2 - 4

1. Two jets are flying towards each other from airports that are $1\ 200$ km apart. One jet is flying at 250 km/h and the other jet at 350 km/h. If they took off at the same time, how long will it take for the jets to pass each other?
2. Kadesh bought 20 shirts at a total cost of R 980. If the large shirts cost R 50 and the small shirts cost R 40. How many of each size did he buy?
3. The diagonal of a rectangle is 25 cm more than its width. The length of the rectangle is 17 cm more than its width. What are the dimensions of the rectangle?
4. The sum of 27 and 12 is equal to 73 more than an unknown number. Find the unknown number.
5. The two smaller angles in a right-angled triangle are in the ratio of $1 : 2$. What are the sizes of the two angles?
6. The length of a rectangle is twice the breadth. If the area is 128 cm^2, determine the length and the breadth.
7. If 4 times a number is increased by 6, the result is 15 less than the square of the number. Find the number.
8. The length of a rectangle is 2 cm more than the width of the rectangle. The perimeter of the rectangle is 20 cm. Find the length and the width of the rectangle.
9. Stephen has $1\ \ell$ of a mixture containing 69% salt. How much water must Stephen add to make the mixture 50% salt? Write your answer as a fraction of a litre.
10. The sum of two consecutive odd numbers is 20 and their difference is 2. Find the two numbers.

11. The denominator of a fraction is 1 more than the numerator. The sum of the fraction and its reciprocal is $\frac{5}{2}$. Find the fraction.
12. Masindi is 21 years older than her daughter, Mulivhu. The sum of their ages is 37. How old is Mulivhu?
13. Tshamano is now five times as old as his son Murunwa. Seven years from now, Tshamano will be three times as old as his son. Find their ages now.

Solutions to Exercise 2 - 4

1. Let distance $d_1 = 1\ 200 - x$ km and $d_2 = x$ km
   speed $S_1 = 250$ km/h and $S_2 = 350$ km/h.
   Time $t = \frac{\text{Distance}}{\text{Speed}}$
   When the jets pass each other $\dfrac{1\ 200 - x}{250} = \dfrac{x}{350}$
   $350(1\ 200 - x) = 250x$
   $420\ 000 - 350x = 250x$
   $600x = 420\ 000$
   $x = 700$ km
   $\therefore t = \dfrac{700 \text{ km}}{350 \text{ km/h}}$
   $= 2$ hours
   It will take take the jets 2 hours to pass each other.

2. Let $x =$ the number of large shirts and $20 - x$ the number of small shirts.
   Populate the data in a table format:

	Nr shirts	Cost	Total
Large	x	50	980
Small	$20 - x$	40	

   $50x + 40(20 - x) = 980$
   $50x + 800 - 40x = 980$
   $10x = 180$
   $\therefore x = 18$
   Kadesh buys 18 large shirts and 2 small shirts.

3. Let length $= l$, width $= w$ and diagonal $= d$.
   $\therefore d = w + 25$ and $l = w + 17$.
   By the theorem of Pythagoras:
   $d^2 = l^2 + w^2$
   $\therefore w^2 = d^2 - l^2$
   $= (w + 25)^2 - (w + 17)^2$
   $= w^2 + 50w + 625 - w^2 - 34w - 289$
   $\therefore w^2 - 16w - 336 = 0$
   $(w + 12)(w - 28) = 0$

$\therefore w = -12$ or $w = 28$
The width must be positive, therefore
width $w = 28$ cm
length $l = (w + 17) = 45$ cm
and diagonal $d = (w + 25) = 53$ cm.

4. Let the unkown number $= x$
$\therefore 27 + 12 = x + 73$
$39 = x + 73$
$x = 39 - 73$
$x = -34$
The unknown number is -34.

5. Let $x =$ the smallest angle. Therefore the other angle $= 2x$.
We are given the third angle $= 90°$.
$x + 2x + 90° = 180°$ (sum of angles in a triangle)
$\therefore 3x = 90°$
$\therefore x = 30°$
The sizes of the angles are $30°$ and $60°$.

6. We are given length $l = 2b$ and $l \times b = 128$
$\therefore 2b \times b = 128$
$2b^2 = 128$
$b^2 = 64$
$b = \pm 8$
But breadth must be positive,
therefore $b = 8$ cm, and $l = 2b = 16$ cm.

7. Let the number $= x$.
Therefore the equation is $4x + 6 = x^2 - 15$.
$\therefore x^2 - 4x - 21 = 0$
$(x - 7)(x + 3) = 0$
$\therefore x = 7$ or $x = -3$

8. Let length $l = x$, width $w = x - 2$ and perimeter $= p$
$\therefore p = 2l + 2w$
$= 2x + 2(x - 2)$
$20 = 2x + 2x - 4$
$4x = 24$
$x = 6$
$\therefore l = 6$ cm and $w = l - 2 = 4$ cm.

9. The new volume (x) of mixture must contain 50% salt, therefore
$0{,}69 = 0.5x$
$\therefore x = \frac{0{,}69}{0{,}5}$
$x = 2(0{,}69) = 1{,}38$
The volume of the new mixture is $1{,}38\ \ell$
The amount of water (y) to be added is
$y = x - 1{,}00$

$= 1{,}38 - 1{,}00$
$= 0{,}38$
Therefore $0{,}38\ \ell$ of water must be added.
To write this as a fraction of a litre:
$0{,}38 = \frac{38}{100}$
$= \frac{19}{50}\ \ell$

10. Let the numbers be x and y.
$x + y = 20$
$x - y = 2$
$2x = 22$
$x = 11$
$11 - y = 2$
$y = 9$
So the numbers are 9 and 11

11. Let the number be x.
So the denominator is $x + 1$.
$\frac{x}{x+1} + \frac{x+1}{x} = \frac{5}{2}$
Solve for x:
$2x^2 + 2(x + 1)^2 = 5x(x + 1)$
$2x^2 + 2x^2 + 4x + 2 = 5x^2 + 5x$
$-x^2 - x + 2 = 0$
$x^2 + x - 2 = 0$
$(x - 1)(x + 2) = 0$
$x = 1$ or $x = -1$
$x + 1 = 2$ or $x + 1 = -1$
So the fraction is $\frac{1}{2}$

12. Let Mulivhu be x years old.
So Masindi is $x + 21$ years old.
$x + x + 21 = 37$
$2x = 16$
$x = 8$
Mulivhu is 8 years old.

13. Let Murunwa be x years old.
So Tshamano is $5x$ years old.
In 7 years time Murunwa's age will be $x + 7$
Tshamano's age will be $5x + 7$
$5x + 7 = 3(x + 7)$
$5x + 7 = 3x + 21$
$2x = 14$
$x = 7$
So Murunwa is 7 years old and Tshamano is 35 years old.

Exercise 2 - 5

1. Make a the subject of the formula: $s = ut + \frac{1}{2}at^2$
2. Solve for n: $pV = nRT$
3. Make x the subject of the formula: $\dfrac{1}{b} + \dfrac{2b}{x} = 2$
4. Solve for r: $V = \pi r^2 h$
5. Solve for h: $E = \dfrac{hc}{\lambda}$
6. Solve for h: $A = 2\pi rh + 2\pi r$
7. Make λ the subject of the formula: $t = \dfrac{D}{f\lambda}$
8. Solve for m: $E = mgh + \frac{1}{2}mv^2$
9. Solve for x: $x^2 + x(a+b) + ab = 0$
10. Solve for b: $c = \sqrt{a^2 + b^2}$
11. Make u the subject of the formula: $\frac{1}{v} = \frac{1}{u} + \frac{1}{w}$
12. Solve for r: $A = \pi R^2 - \pi r^2$
13. $F = \frac{9}{5}C + 32°$ is the formula for converting temperature in degrees Celsius to degrees Fahrenheit. Derive a formula for converting degrees Fahrenheit to degrees Celsius.
14. $V = \frac{4}{3}\pi r^3$ is the formula for determining the volume of a soccer ball. Express the radius in terms of the volume.

Solutions to Exercise 2 - 5

1. $s = ut + \frac{1}{2}at^2$
   $s - ut = \frac{1}{2}at^2$
   $2(s - ut) = at^2$
   $\therefore a = \dfrac{2(s - ut)}{t^2}$
2. $pV = nRT$
   $\therefore n = \dfrac{pV}{RT}$
3. $\dfrac{1}{b} + \dfrac{2b}{x} = 2$
   $\dfrac{2b}{x} = 2 - \dfrac{1}{b}$

$\frac{2b}{x} = \frac{2b-1}{b}$
$\therefore x(2b-1) = 2b^2$
$x = \frac{2b^2}{2b-1}$

4. $V\pi r^2 h$
$r^2 = \frac{V}{\pi h}$
$\therefore V = \sqrt{\frac{V}{\pi h}}$

5. $E = \frac{hc}{\lambda}$
$E\lambda = hc$
$\therefore h = \frac{E\lambda}{c}$

6. $A = 2\pi rh + 2\pi r$
$2\pi rh = A - 2\pi r$
$\therefore h = \frac{A - 2\pi r}{2\pi r}$

7. $t = \frac{D}{f\lambda}$
$ft\lambda = D$
$\therefore \lambda = d\frac{D}{ft}$

8. $E = mgh + \frac{1}{2}mV^2$
$E = m\left(gh + \frac{1}{2}V^2\right)$
$\therefore m = \frac{E}{gh + \frac{1}{2}V^2}$

9. $x^2 + x(a+b) + ab = 0$
$(x+a)(x-b) = 0$
$x = -a$ or $x = -b$

10. $c = \sqrt{a^2 + b^2}$
$c^2 = a^2 + b^2$
$c^2 - a^2 = b^2$
$b = \pm\sqrt{c^2 - a^2}$

11. $\frac{1}{v} = \frac{1}{u} + \frac{1}{w}$
$uw - uv = vw$
$u(w - v) = vw$
$u = \frac{vw}{w-v}$

12. $A = \pi R^2 - \pi r^2$
$A = \pi(R^2 - r^2)$
$\frac{A}{\pi} = R^2 - r^2$
$r^2 = R^2 - \frac{A}{\pi}$
$r = \pm\sqrt{R^2 - \frac{A}{\pi}}$

13. $F = \frac{9}{5}C + 32$
$5F = 9C + 160$
$9C = 160 - 5F$
$C = \frac{160}{9} - \frac{5}{9}F$

14. $V = \frac{4}{3}\pi r^3$
$3V = 4\pi r^3$
$r^3 = \frac{3V}{4\pi}$
$r = \sqrt[3]{\frac{3V}{4\pi}}$

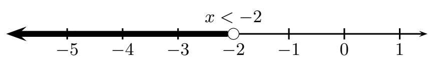
Exercise 2 - 6

Solve for x and represent the answer on a number line and in interval notation:

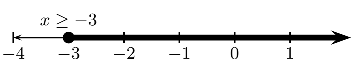
1. $3x + 4 > 5x + 8$
2. $3(x-1) - 2 \leq 6x + 4$
3. $\dfrac{x-7}{3} > \dfrac{2x-3}{2}$
4. $-4(x-1) < x + 2$
5. $\dfrac{1}{2}x + \dfrac{1}{3}(x-1) \geq \dfrac{5}{6}x - \dfrac{1}{3}$
6. $-2 \leq x - 1 < 3$
7. $-5 < 2x - 3 \leq 7$
8. $7(3x+2) - 5(2x-3) > 7$
9. $\frac{5x-1}{-6} \leq \frac{1-2x}{3}$
10. $3 \leq 4 - x \leq 16$
11. $\frac{-7y}{3} - 5 > -7$
12. $1 \leq 1 - 2y < 9$
13. $-2 < \frac{x-1}{-3} < 7$

Solutions to Exercise 2 - 6

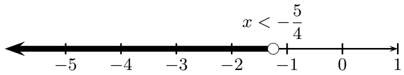
1. $3x + 4 > 5x + 8$
$3x - 5x > 8 - 4$
$-2x > 4$
$2x < -4$
$x < -2$



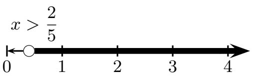
2. $3(x-1) - 2 \leq 6x + 4$
$3x - 3 - 2 \leq 6x + 4$
$3x - 6x \leq 4 + 5$
$-3x \leq 9$
$3x \geq -9$
$x \geq -3$



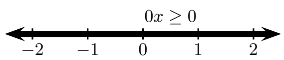
3. $\frac{x-7}{3} > \frac{2x-3}{2} 2(x-7) > 3(2x-3)$
$2x - 14 > 6x - 9$
$-4x > 5$
$x < -\frac{5}{4}$



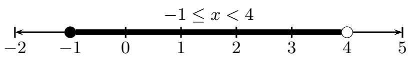
4. $-4(x-1) < x+2$
$-4x + 4 < x + 2$
$-5x < -2$
$x > \frac{2}{5}$



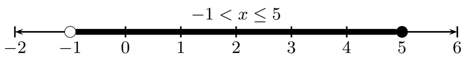
5. $\frac{1}{2}x + \frac{1}{3}(x-1) \geq \frac{5}{6}x - \frac{1}{3}$
$3x + 2(x-1) \geq 5x - 2$
$3x + 2x - 2 \geq 5x - 2$
$5x - 5x \geq 2 - 2$
$0x \geq 0$
This is true for all real values of x.



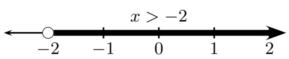
6. $-2 \leq x - 1 < 3$
$-2 + 1 \leq x - 1 + 1 < 3 + 1$
$-1 \leq x < 4$



7. $-5 < 2x - 3 \leq 7 - 5 + 3 < 2x - 3 \leq 7 + 3$
$-2 < 2x \leq 10$
$-1 < x \leq 5$



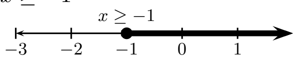
8. $7(3x+2) - 5(2x-3) > 7$
$21x + 14 - 10x + 15 > 7$
$11x > 7 - 14 - 15$
$11x > -22$
$x > -2$



9. $\frac{5x-1}{-6} \leq \frac{1-2x}{3}$
$5x - 1 \geq -2(1-2x)$
$5x - 1 \geq -2 + 4x$

$5x - 4x \geq -1$

$x \geq -1$

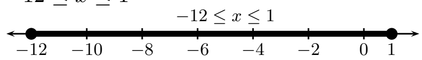


10. $3 \leq 4 - x \leq 16$

$-1 \leq -x \leq 12$

$1 \geq x \geq -12$

$-12 \leq x \leq 1$

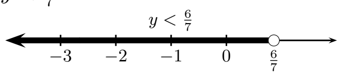


11. $\frac{-7y}{3} - 5 > -7$

$-7y - 15 > -21$

$-7y > -6$

$y < \frac{6}{7}$

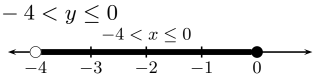


12. $1 \leq 1 - 2y < 9$

$0 \leq -2y < 8$

$0 \geq y > -4$

$-4 < y \leq 0$

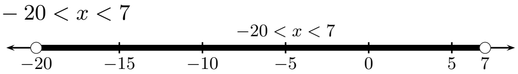


13. $-2 < \frac{x-1}{-3} < 7$

$6 > x - 1 > -21$

$7 > x > -20$

$-20 < x < 7$



End of Chapter Exercises

1. Solve:

(a) $2(p-1) = 3(p+2)$

(b) $3 - 6k = 2k - 1$

(c) $m + 6(-m+1) + 5m = 0$

(d) $2k + 3 = 2 - 3(k+3)$

(e) $5t - 1 = t^2 - (t+2)(t-2)$

(f) $3 + \dfrac{q}{5} = \dfrac{q}{2}$

(g) $5 - \dfrac{2(m+4)}{m} = \dfrac{7}{m}$

(h) $\dfrac{2}{t} - 2 - \dfrac{1}{2} = \dfrac{1}{2}\left(1 + \dfrac{2}{t}\right)$

(i) $x^2 - 3x + 2 = 0$

(j) $y^2 + y = 6$

(k) $0 = 2x^2 - 5x - 18$

(l) $(d+4)(d-3) - d = (3d-2)^2 - 8d(d-1)$

(m) $5x + 2 \leq 4(2x-1)$

(n) $\dfrac{4x-2}{6} > 2x + 1$

(o) $\frac{x}{3} - 14 > 14 - \frac{x}{7}$

(p) $\frac{1-a}{2} - \frac{2-a}{3} \geq 1$

(q) $-5 \leq 2k + 1 < 5$

(r) $x - 1 = \frac{42}{x}$

2. Consider the following literal equations:

(a) Solve for I: $P = VI$

(b) Make m the subject of the formula: $E = mc^2$

(c) Solve for t: $v = u + at$

(d) Make f the subject of the formula: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

(e) Make C the subject of the formula: $F = \frac{9}{5}C + 32$

(f) Solve for y: $m = \frac{y-c}{x}$

3. Solve the following simultaneous equations:

(a) $7x + 3y = 13$ and $2x - 3y = -4$

(b) $10 = 2x + y$ and $y = x - 2$

(c) $7x - 41 = 3y$ and $17 = 3x - y$

(d) $2y = x + 8$ and $4y = 2x - 44$

4. Find the solutions to the following word problems:

(a) $\frac{7}{8}$ of a certain number is 5 more than of $\frac{1}{3}$ of the number. Find the number.

(b) Three rulers and two pens have a total cost of R $21{,}00$. One ruler and one pen have a total cost of R $8{,}00$. How much does a ruler cost and how much does a pen cost?

(c) A man runs to the bus stop and back in 15 minutes. His speed on the way to the bus stop is 5 km/h and his speed on the way back is 4 km/h. Find the distance to the bus stop.

(d) Zanele and Piet skate towards each other on a straight path. They set off 20 km apart. Zanele skates at 15 km/h and Piet at 10 km/h. How far will Piet have skated when they reach each other?

(e) When the price of chocolates is increased by R 10, we can buy five fewer chocolates for R 300. What was the price of each chocolate before the price was increased?

Solutions to End of Chapter Exercises

1. **(a)** $2(p-1) = 3(p+2)$
$2p - 2 = 3p + 6$
$-p = 8$
$\therefore\ p = -8$

(b) $3 - 6k = 2k - 1$
$-8k = -4$
$\therefore\ k = \frac{-4}{-8} = \frac{1}{2}$

(c) $m + 6(-m + 1) + 5m = 0$
$m - 6m + 6 + 5m = 0$
$0m = -6$
No solution

(d) $2k + 3 = 2 - 3(k + 3)2k + 3 =$
$2 - 3k - 9$
$5k = -10$

$\therefore\ k = \frac{-10}{5} = -2$

(e) $5t - 1 = t^2 - (t+2)(t-2)$
$= 5t - 1 = t^2 - t^2 + 4$
$5t = 5$
$t = 1$

(f) $3 + \frac{q}{5} = \frac{q}{2}$
$30q + 2q = 5q$
$3q = 30$
$q = 10$

(g) $5 - \frac{2(m+4)}{m} = \frac{7}{m}$
$5m - 2m - 8 = 7$
$3m = 15$
$m = 5$

(h) $\frac{2}{t} - 2 - \frac{1}{2} = \frac{1}{2}\left(1 + \frac{2}{t}\right)$
$\frac{2}{t} - 2 - \frac{1}{2} = \frac{1}{2} + \frac{1}{t}$
$\frac{2}{t} - \frac{1}{t} = 1 + 2$
$\frac{1}{t} = 3$
$\therefore t = \frac{1}{3}$

(i) $x^2 - 3x + 2 = 0$
$(x-2)(x-1) = 0$
$\therefore x = 2$ or $x = 1$

(j) $y^2 + y = 6$
$y^2 + y - 6 = 0$
$(y+3)(y-2) = 0$
$\therefore y = -3$ or $y = 2$

(k) $0 = 2x^2 - 5x - 18$
$(2x+9)(x-2) = 0$
$\therefore x = -\frac{9}{2}$ or $x = 2$

(l) $(d+4)(d-3) - d = (3d-2)^2 - 8d(d-1)$
$d^2 + d - 12 - d = 9d^2 - 12d + 4 - 8d^2 + 8d$
$d^2 - 12 = d^2 - 4d + 4$
$4d = 16$
$\therefore d = 4$

(m) $5x + 2 \leq 4(2x-1)$
$5x - 8x \leq -4 - 2$
$-3x \leq -6$
$\therefore\ x \geq 2$

(n) $\dfrac{4x-2}{6} > 2x + 1$
$4x - 2 > 12x + 6$
$4x - 12x > 6 + 2$
$-8x > 8$
$\therefore\ x < -1$

(o) $\frac{x}{3} - 14 > 14 - \frac{x}{7}$
$7x - 294 > 294 - 3x$
$10x > 588$
$\therefore x > \frac{588}{10}$

(p) $\dfrac{1-a}{2} - \dfrac{2-a}{3} \geq 1$
$\dfrac{1-a}{2} - \dfrac{(2+a)}{3} \geq 1$
$3 - 3a - 4 - 2a \geq 6$
$-5a \geq 7$
$\therefore\ a \leq -\frac{7}{5}$

(q) $-5 \leq 2k + 1 < 5$
$-6 \leq 2k < 4$
$\therefore -3 \leq k < 2$

(r) $x - 1 = \dfrac{42}{x}$
Note that $x \neq 0$.
$x^2 - x = 42$
$x^2 - x - 42 = 0$
$(x-7)(x+6) = 0$
$\therefore x = 7$ or $x = -6$

2. **(a)** $P = VI$
$\therefore I = \frac{P}{V}$

(b) $E = mc^2$
$\therefore m = \dfrac{e}{c^2}$

(c) $v = u + at$
$\therefore t = \dfrac{v-u}{a}$

(d) $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$
$\dfrac{v+u}{uv} = \dfrac{1}{f}$
$f(v+u) = uv$

$\therefore f = \dfrac{uv}{u+v}$

(e) $F = \frac{9}{5}C + 32$

$\therefore C = \frac{5}{9}(F - 32)$

(f) $m = \dfrac{y-c}{x}$

$\therefore y = mx + c$

3. **(a)** $7x + 3y = 13$ and $2x - 3y = -4$

Add the two equations together to solve for x:

$$\begin{array}{rrcl} & 7x + 3y & = & 13 \\ + & 2x - 3y & = & -4 \\ \hline & 9x + 0 & = & 9 \end{array}$$

$\therefore x = 1$

Substitute value of x into second equation:

$2(1) - 3y = -4$

$-3y = -6$

$y = 2$

(b) $10 = 2x + y$ and $y = x - 2$

Substitute value of y into first equation:

$10 = 2x + (x - 2)$

$3x = 12$

$\therefore x = 4$

Substitute value of x into second equation:

$y = 4 - 2$

$= 2$

(c) $7x - 41 = 3y$ and $17 = 3x - y$

$17 = 3x - y$

$\therefore y = 3x - 17$

Substitute the value of y into first equation:

$7x - 41 = 3(3x - 17)$

$7x - 41 = 9x - 51$

$2x = 10$

$\therefore x = 5$

Substitute value of x into second equation:

$y = 3(5) - 17$

$= -2$

(d) $2y = x + 8$ and $4y = 2x - 44$

$2y = x + 8$

$\therefore x = 2y - 8$

Substitute value of x into second equation:

$4y = 2(2y - 8) - 44$

$4y = 4y - 16 - 44$

$\therefore 0y = -60$

No solution

($y_1 = \frac{1}{2}x - 11$ and $y_2 = \frac{1}{2}x + 4$. These lines have the same gradient therefore they are parallel and will never intersect - hence there is no solution.)

4. (a) Let x be the number.
$\therefore\ \frac{7}{8}x = \frac{1}{3}x + 5$
$21x = 8x + 120$
$13x = 120$
$x = \frac{120}{13}$

(b) Let the price of rulers be x and the price of pens be y
$\therefore 3x + 2y = 21$ and $x + y = 8$
From the second equation: $x = 8 - y$
Substitute the value of x into the first equation:
$3(8 - y) + 2y = 21$
$24 - 3y + 2y = 21$
$-y = -3$
$\therefore y = 3$
Substitute the value of y into the second equation:
$x + 3 = 8$
$\therefore x = 5$
Therefore each ruler costs R 5 and each pen costs R 3.

(c) Let x be the distance to the bus.
Speed $s_1 = 5$ km/h and $s_2 = 4$ km/h
Distance $=$ speed $\times$ time $= s \times t$
$\therefore x = 5 \times t_1 = 4 \times t_1$
$t_1 + t_2 = 15$ minutes $= 0{,}25$hours
$t_1 = \frac{x}{5}$ and $t_2 = \frac{x}{4}$
$\therefore\ \frac{x}{5} + \frac{x}{4} = 0{,}25$
$4x + 5x = 0{,}25 \times 20$
$9x = 5$
$\therefore x = \frac{5}{9}$ km
The bus stop is $\frac{5}{9}$ km or $0{,}56$ km away.

(d) Let x be the distance Zanele skates and $20 - x$ the distance Piet skates.
Populate the data in a table:

	Speed	Distance	Time
Zanele	15	x	$\frac{x}{15}$
Piet	$20 - x$	$20 - x$	$\frac{20-x}{10}$

$\dfrac{x}{15} = \dfrac{20 - x}{10}$
$10x = 15(20 - x)$
$10x = 300 - 15x$
$25x = 300$
$\therefore x = 12$
$\therefore y = 20 - 12 = 8$
Zanele will have skated 12 km and Piet will have skated 8 km when they reach each other.

(e) Let x be the original price of chocolates.
New price $\times$ number of chocolates $= 300$
$\therefore (x + 10)(\frac{300}{x} - 5) = 300$
$300 - 5x + \frac{3000}{x} - 50 = 300$

$-5x + \frac{3000}{x} - 50 = 0$
$-5x^2 + 3000 - 50x = 0$
$x^2 + 10x - 600 = 0$
$(x - 20)(x + 30) = 0$
$\therefore x = 20$ or $x = -30$
Price must be positive $\therefore x = 20$.
The price of each chocolate before the price increase was R 20.

Exponents 3

Exercise 3 - 1

Simplify without using a calculator:

1. 16^0
2. $16a^0$
3. $\frac{2^{-2}}{3^2}$
4. $\frac{5}{2^{-3}}$
5. $\left(\frac{2}{3}\right)^{-3}$
6. x^2x^{3t+1}
7. $3 \times 3^{2a} \times 3^2$
8. $\frac{a^{3x}}{a^x}$
9. $\frac{32p^2}{4p^8}$
10. $(2t^4)^3$
11. $(3^{n+3})^2$
12. $\frac{3^n 9^{n-3}}{27^{n-1}}$

Solutions to Exercise 3 - 1

1. 1

2. $16a^0$
$= 16(1)$
$= 16$

3. $\frac{2^{-2}}{3^2}$
$= \frac{1}{2^2.3^2}$
$= \frac{1}{4 \times 9}$
$= \frac{1}{36}$

4. $\frac{5}{2^{-3}}$
$= (5)(2^3)$
$= (5)(8)$
$= 40$

5. $\left(\frac{2}{3}\right)^{-3}$
$= \frac{2^{-3}}{3^{-3}}$
$= \frac{3^3}{2^3}$
$= \frac{27}{8}$

6. x^2x^{3t+1}
$= x^{3t+3}$

7. $3 \times 3^{2a} \times 3^2$
$= 3^{1+2a+2}$
$= 3^{2a+3}$

8. $\frac{a^{3x}}{a^x}$
$= a^{3x}.a^{-x}$
$= a^{3x-x}$

$= a^{2x}$

9. $\frac{32p^2}{4p^8}$
$= 8p^{2-8}$
$= 8p^{-6}$
$= \frac{8}{p^6}$

10. $(2t^4)^3$
$= 2^3.t^{4.3}$
$= 8t^{12}$

11. $(3^{n+3})^2$
$= 3^{2n+6}$

12. $\frac{3^n 9^{n-3}}{27^{n-1}}$
$= \frac{3^n.(3^2)^{n-3}}{(3^3)^{n-1}}$
$= \frac{3^n.3^{2n-6}}{3^{3n-3}}$
$= 3^{n+2n-6-(3n-3)}$
$= 3^{3n-6-3n+3}$
$= 3^{-3}$
$= \frac{1}{3^3}$
$= \frac{1}{27}$

Exercise 3 - 2

Simplify without using a calculator:

1. $t^{\frac{1}{4}} \times 3t^{\frac{7}{4}}$
2. $\frac{16x^2}{(4x^2)^{\frac{1}{2}}}$
3. $(0{,}25)^{\frac{1}{2}}$
4. $(27)^{-\frac{1}{3}}$
5. $(3p^2)^{\frac{1}{2}} \times (3p^4)^{\frac{1}{2}}$

Solutions to Exercise 3 - 2

1. $t^{\frac{1}{4}} \times 3t^{\frac{7}{4}}$
$= 3t^{(\frac{1}{4}+\frac{7}{4})}$
$= 3t^{\frac{8}{4}}$
$= 3t^2$

2. $\frac{16x^2}{(4x^2)^{\frac{1}{2}}}$
$= \frac{16x^2}{4^{\frac{1}{2}}x^{2\frac{1}{2}}}$
$= \frac{4^2x^2}{4^{\frac{1}{2}}x}$
$= 4^{2-\frac{1}{2}}.x^{2-1}$
$= 4^{\frac{3}{2}}.x$
$= (2^2)^{\frac{3}{2}}x$
$= 2^3x$
$= 8x$

3. $(0{,}25)^{\frac{1}{2}}$
$= \left(\frac{1}{4}\right)^{\frac{1}{2}}$
$= \left(\frac{1}{2^2}\right)^{\frac{1}{2}}$
$= \left(2^{-2}\right)^{\frac{1}{2}}$
$= 2 - 1$
$= \frac{1}{2}$

4. $(27)^{-\frac{1}{3}}$
$= (3^3)^{-\frac{1}{3}}$
$= 3^{-1}$
$= \frac{1}{3}$

5. $(3p^2)^{\frac{1}{2}} \times (3p^4)^{\frac{1}{2}}$
$= 3^{\frac{1}{2}}p \times 3^{\frac{1}{2}}p^2$
$= 3^{\frac{1}{2}+\frac{1}{2}} \times p^{1+2}$
$= 3p^3$

Exercise 3 - 3

1. Solve for the variable:

(a) $2^{x+5} = 32$

(b) $5^{2x+2} = \frac{1}{125}$

(c) $64^{y+1} = 16^{2y+5}$

(d) $3^{9x-2} = 27$

(e) $81^{k+2} = 27^{k+4}$

(f) $25^{(1-2x)} - 5^4 = 0$

(g) $27^x \times 9^{x-2} = 1$

(h) $2^t + 2^{t+2} = 40$

(i) $2 \times 5^{2-x} = 5 + 5^x$

(j) $9^m + 3^{3-2m} = 28$

(k) $y - 2y^{\frac{1}{2}} + 1 = 0$

(l) $4^{x+3} = 0{,}5$

(m) $2^a = 0{,}125$

(n) $10^x = 0{,}001$

(o) $2^{x^2-2x-3} = 1$

2. The growth of algae can be modelled by the function $f(t) = 2^t$. Find the value of t such that $f(t) = 128$.

Solutions to Exercise 3 - 3

1. (a)
$$2^{x+5} = 32$$
$$2^{x+5} = 2^5$$
$$x + 5 = 5$$
$$x = 0$$

(b)
$$5^{2x+2} = \frac{1}{125}$$
$$5^{2x+2} = \frac{1}{5^3}$$
$$5^{2x+2} = 5^{-3}$$
$$2x + 2 = -3$$
$$2x = -5$$
$$x = -\tfrac{5}{2}$$

(c)
$$64^{y+1} = 16^{2y+5}$$
$$2^{6(y+1)} = 2^{4(2y+5)}$$
$$2^{6y+6} = 2^{8y+20}$$
$$6y + 6 = 8y + 20$$
$$6y - 8y = 20 - 6$$
$$-2y = 14$$
$$y = -7$$

(d)
$$3^{9x-2} = 27$$
$$3^{9x-2} = 3^3$$
$$9x - 2 = 3$$
$$9x = 5$$
$$x = \tfrac{5}{9}$$

(e)
$$81^{k+2} = 27^{(k+4)}$$
$$3^{4(k+2)} = 3^{3(k+4)}$$
$$3^{4k+8} = 3^{3k+12}$$
$$4k + 8 = 3k + 12$$
$$4k - 3k = 12 - 8$$
$$k = 4$$

(f)
$$25^{(1-2x)} - 5^4 = 0$$
$$5^{2(1-2x)} = 5^4$$
$$2(1 - 2x) = 4$$

$1-2x=2$
$-2x=1$
$x=-\frac{1}{2}$

(g) $27^x \times 9^{x-2}=1$
$3^{3x}\times 3^{2(x-2)}=1$
$3^{3x+2(x-2)}=3^0$
$3x+2x-4=0$
$5x-4=0$
$5x=4$
$x=\dfrac{4}{5}$

(h) $2^t+2^{t+2}=40$
$2^t+2^t.2^2=40$
$2^t(1+2^2)=40$
$2^t(1+4)=40$
$2^t=\dfrac{40}{(1+4)}$
$2^t=\dfrac{40}{5}$
$2^t=8$
$2^t=2^3$
$t=3$

(i) $2.5^{2-x}=5+5^x$
$2.5^2.5^{-x}=5+5^x$
$\dfrac{2.5^2}{5^x}-5-5^x=0$
$\left(\frac{50}{5^x}\right)\times 5^x-5\times(5^x)-5^x\times(5^x)=0$
$50-5.5^x-(5^x)^2=0$
$(5^x)^2+5.5^x-50=0$
$(5^x-5)(5^x+10)=0$
$5^x-5=0$ or $5^x+10=0$
$5^x=5$ or $5^x=-10$
if $5^x=-10:$ no solution
if $5^x=5:$
$5^x=5^1$
$x=1$

(j) $9^m+3^{3-2m}=28$
$3^{2m}+3^3.3^{-2m}=28$
$3^{2m}+\dfrac{27}{3^{2m}}-28=0$
$(3^{2m})^2-28(3^{2m})+27=0$
$(3^{2m}-27)(3^{2m}-1)=0$
$3^{2m}-27=0$ or $3^{2m}-1=0$
$3^{2m}=27$ or $3^{2m}=1$
if $3^{2m}=27$
$3^{2m}=3^3$
$2m=3$
$m=\frac{3}{2}$
if $3^{2m}=1$
$3^{2m}=3^0$
$2m=0$
$m=0$

(k) $y-2y^{\frac{1}{2}}+1=0$
$(y^{\frac{1}{2}}-1)(y^{\frac{1}{2}}-1)=0$
$y^{\frac{1}{2}}-1=0$
$y^{\frac{1}{2}}=1$
$y^{\frac{1}{2}}=1^{\frac{1}{2}}$
$y=1$

(l) $4^{x+3}=0{,}5$
$2^{2x+6}=2^{-1}$
$2x+6=-1$
$2x=-7$
$x=-3\frac{1}{2}$

(m) $2^a=0{,}125$
$2^a=\frac{125}{1000}$
$2^a=\frac{1}{8}$
$2^a=\frac{1}{2^3}$
$2^a=2^{-3}$
$a=-3$

(n) $10^x=0{,}001$
$10^x=\frac{1}{1000}$
$=\frac{1}{10^3}$
$=10^{-3}$
$x=-3$

(o) $2^{x^2-2x-3}=1$
$2^{x^2-2x-3}=2^0$
$x^2-2x-3=0$
$(x-3)(x+1)=0$
$x=3$ or $x=-1$

2. $2^t=128$
$2^t=2^7$
$t=7$

End of Chapter Exercises

1. Simplify:

(a) $t^3 \times 2t^0$

(b) $5^{2x+y}5^{3(x+z)}$

(c) $(b^{k+1})^k$

(d) $\dfrac{6^{5p}}{9^p}$

(e) $m^{-2t} \times (3m^t)^3$

(f) $\dfrac{3x^{-3}}{(3x)^2}$

(g) $\dfrac{5^{b-3}}{5^{b+1}}$

(h) $\dfrac{2^{a-2}3^{a+3}}{6^a}$

(i) $\dfrac{3^n 9^{n-3}}{27^{n-1}}$

(j) $\left(\dfrac{2x^{2a}}{y^{-b}}\right)^3$

(k) $\dfrac{2^{3x-1}8^{x+1}}{4^{2x-2}}$

(l) $\dfrac{6^{2x}11^{2x}}{22^{2x-1}3^{2x}}$

(m) $\dfrac{(-3)^{-3}(-3)^2}{(-3)^{-4}}$

(n) $(3^{-1}+2^{-1})^{-1}$

(o) $\dfrac{9^{n-1}27^{3-2n}}{81^{2-n}}$

(p) $\dfrac{2^{3n+2}8^{n-3}}{4^{3n-2}}$

(q) $\dfrac{3^{t+3}+3^t}{2 \times 3^t}$

(r) $\dfrac{2^{3p}+1}{2^p+1}$

2. Solve:

(a) $3^x = \dfrac{1}{27}$

(b) $5^{t-1} = 1$

(c) $2 \times 7^{3x} = 98$

(d) $2^{m+1} = (0{,}5)^{m-2}$

(e) $3^{y+1} = 5^{y+1}$

(f) $z^{\frac{3}{2}} = 64$

(g) $16x^{\frac{1}{2}} - 4 = 0$

(h) $m^0 + m^{-1} = 0$

(i) $t^{\frac{1}{2}} - 3t^{\frac{1}{4}} + 2 = 0$

(j) $3^p + 3^p + 3^p = 27$

(k) $k^{-1} - 7k^{-\frac{1}{2}} - 18 = 0$

(l) $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$

Solutions to End of Chapter Exercises

1. **(a)** $t^3 \times 2t^0$
$= t^3 \times 2(1)$
$= 2t^3$

(b) $5^{2x+y} \times 5^{3(x+z)}$
$= 5^{2x+y+3x+3z}$
$= 5^{5x+y+3z}$

(c) $\left(b^{k+1}\right)^k = b^{k^2+k}$

(d) $\dfrac{6^{5p}}{9^p}$
$= \dfrac{2^{5p} \times 3^{5p}}{3^{2p}}$
$= 2^{5p} \times 3^{5p-2p}$
$= 2^{5p} \times 3^{3p}$
$= \left(2^5 \times 3^3\right)^p$
$= (32 \times 27)^p$
$= 864^p$

(e) $m^{-2t} \times \left(3m^t\right)^3$
$= m^{-2t} \times 3^3 m^{3t}$
$= m^{-2t+3t} \times 27$
$= 27m^t$

(f) $\dfrac{3x^{-3}}{(3x)^2}$
$= 3^{1-2} \times x^{-3-2}$
$= 3^{-1} \times x^{-5}$
$= \dfrac{1}{3x^5}$

(g) $\dfrac{5^{b-3}}{5^{b+1}}$
$= 5^{b-3-b-1}$
$= 5^{-4}$
$= \dfrac{1}{5^4}$
$= \dfrac{1}{625}$

(h) $\dfrac{2^{a-2}3^{a+3}}{6^a}$
$= \dfrac{2^{a-2} \times 3^{a+3}}{(2 \times 3)^a}$
$= \dfrac{2^{a-2} \times 3^{a+3}}{2^a \times 3^a}$
$= 2^{a-2-a} \times 3^{a+3-a}$
$= 2^{-2} \times 3^3$
$= \dfrac{3^3}{2^2}$
$= \dfrac{27}{4}$

(i) $\dfrac{3^n 9^{n-3}}{27^{n-1}}$
$= \dfrac{3^n \times \left(3^2\right)^{n-3}}{\left(3^3\right)^{n-1}}$
$= \dfrac{3^n \times 3^{2n-6}}{3^{3n-3}}$
$= 3^{n+2n-6-(3n-3)}$
$= 3^{3n-6-3n+3}$
$= 3^{-3}$
$= \dfrac{1}{3^3}$
$= \dfrac{1}{27}$

(j) $\left(\dfrac{2x^{2a}}{y^{-b}}\right)^3$
$= \dfrac{2^3 \left(x^{2a}\right)^3}{\left(y^{-b}\right)^3}$
$= \dfrac{2^3 x^{6a}}{y^{-3b}}$
$= 2^3 x^{6a} y^{3b}$
$= 8x^{6a}y^{3b}$

(k) $\dfrac{2^{3x-1}8^{x+1}}{4^{2x-2}}$
$= \dfrac{2^{3x-1} \times 2^{3(x+1)}}{2^{2(2x-2)}}$
$= 2^{3x-1+3x+3-4x+4}$
$= 2^{2x+6}$
$= 4^{x+3}$

(l) $\dfrac{6^{2x}11^{2x}}{22^{2x-1}3^{2x}}$
$= \dfrac{(3 \times 2)^{2x} \times 11^{2x}}{(2 \times 11)^{2x-1} \times 3^{2x}}$
$= \dfrac{3^{2x} \times 2^{2x} \times 11^{2x}}{2^{2x-1} \times 11^{2x-1} \times 3^{2x}}$
$= 3^{2x-2x} \times 2^{2x-2x+1} \times 11^{2x-2x+1}$
$= 3^0 \times 2^1 \times 11^1$
$= 1 \times 2 \times 11$
$= 22$

(m) $\dfrac{(-3)^{-3}(-3)^2}{(-3)^{-4}}$
$= (-3)^{-3+2+4}$
$= (-3)^3$
$= -27$

(n) $(3^{-1} + 2^{-1})^{-1}$
$= \left(\dfrac{1}{3} + \dfrac{1}{2}\right)^{-1}$
$= \left(\dfrac{2}{6} + \dfrac{3}{6}\right)^{-1}$
$= \left(\dfrac{5}{6}\right)^{-1}$
$= \left(\dfrac{5^{-1}}{6^{-1}}\right)$
$= \dfrac{6}{5}$

(o) $\dfrac{9^{n-1}27^{3-2n}}{81^{2-n}}$
$= \dfrac{3^{2(n-1)} \times 3^{3(3-2n)}}{3^{4(2-n)}}$
$= 3^{2(n-1)+3(3-2n)-4(2-n)}$
$= 3^{2n-2+9-6n-8+4n}$
$= 3^{-1}$
$= \dfrac{1}{3}$

(p) $\dfrac{2^{3n+2}8^{n-3}}{4^{3n-2}}$
$= \dfrac{2^{3n+2} \times 2^{3(n-3)}}{2^{2(3n-2)}}$
$= 2^{3n+2+3(n-3)-2(3n-2)}$

$= 2^{3n+2+3n-9-6n+4}$
$= 2^{-3}$
$= \frac{1}{2^3}$
$= \frac{1}{8}$

(q) $\frac{3^{t+3}+3^t}{2\times 3^t}$
$= \frac{3^t\times 3^3+3^t}{2\times 3^t}$
$= \frac{3^t(3^3+1)}{2\times 3^t}$
$= \frac{3^3+1}{2}$
$= \frac{28}{2}$
$= 14$

(r) $\frac{2^{3p}+1}{2^p+1}$
$= \frac{(2^p+1)(2^{2p}-2^p+1)}{(2^p+1)}$
$= 2^{2p}-2^p+1$

2. **(a)** $3^x = \frac{1}{27}$
$3^x = \frac{1}{3^3}$
$3^x = 3^{-3}$
$x = -3$

(b) $5^{t-1} = 1$
$5^{t-1} = 5^0$
$t-1 = 0$
$t = 1$

(c) $2\times 7^{3x} = 98$
$7^{3x} = 49$
$7^{3x} = 7^2$
$3x = 2$
$x = \frac{2}{3}$

(d) $2^{m+1} = (0{,}5)^{m-2}$
$2^{m+1} = \left(\frac{1}{2}\right)^{m-2}$
$2^{m+1} = \left(2^{-1}\right)^{m-2}$
$2^{m+1} = (2)^{2-m}$
$m+1 = 2-m$
$2m = 1$
$m = \frac{1}{2}$

(e) $3^{y+1} = 5^{y+1}$
$y+1 = 0$
$y = -1$

(f) $z^{\frac{3}{2}} = 64$
$z^{\frac{3}{2}} = 4^3$
$\left(z^{\frac{3}{2}}\right)^{\frac{2}{3}} = (4^3)^{\frac{2}{3}}$
$z = 4^2$
$z = 16$

(g) $16x^{\frac{1}{2}} - 4 = 0$
$16x^{\frac{1}{2}} = 4$
$x^{\frac{1}{2}} = \frac{4}{16}$
$x^{\frac{1}{2}} = \frac{1}{4}$
$\left(x^{\frac{1}{2}}\right)^2 = \left(\frac{1}{4}\right)^2$
$x = \frac{1}{16}$

(h) $m^0 + m^{-1} = 0$
$1 + m^{-1} = 0$
$m^{-1} = -1$
$\left(m^{-1}\right)^{-1} = (-1)^{-1}$
$m = -1$

(i) $t^{\frac{1}{2}} - 3t^{\frac{1}{4}} + 2 = 0$
$(t^{\frac{1}{4}}-1)(t^{\frac{1}{4}}-2)$
$t^{\frac{1}{4}} - 1 = 0$ or $t^{\frac{1}{4}} - 2 = 0$
if $t^{\frac{1}{4}} - 1 = 0:$
$t^{\frac{1}{4}} = 1$
$\left(t^{\frac{1}{4}}\right)^4 = 1^4$
$t = 1$
if $t^{\frac{1}{4}} - 2 = 0:$
$t^{\frac{1}{4}} = 2$
$\left(t^{\frac{1}{4}}\right)^4 = 2^4$
$t = 16$

(j) $3^p + 3^p + 3^p = 27$
$3\times 3^p = 27$
$3^{p+1} = 3^3$
$p+1 = 3$
$p = 2$

(k) $k^{-1} - 7k^{-\frac{1}{2}} - 18 = 0$
$(k^{-\frac{1}{2}}-9)(k^{-\frac{1}{2}}+2) = 0$
$k^{-\frac{1}{2}} - 9 = 0$ or $k^{-\frac{1}{2}} + 2 = 0$
$k^{-\frac{1}{2}} = 9$ or $k^{-\frac{1}{2}} = -2$
if $k^{-\frac{1}{2}} = 9:$
$(k^{-\frac{1}{2}})^{-2} = 9^{-2}$
$k = \frac{1}{81}$
if $k^{-\frac{1}{2}} = -2:$
$(k^{-\frac{1}{2}})^{-2} = (-2)^{-2}$
$k = \frac{1}{4}$

(l) $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$

$(x^{\frac{1}{4}}+6)(x^{\frac{1}{4}}-3)=0$
$x^{\frac{1}{4}}+6=0$ or $x^{\frac{1}{4}}-3=0$
$x^{\frac{1}{4}}=-6$ or $x^{\frac{1}{4}}=3$
if $x^{\frac{1}{4}}=-6:$
$\left(x^{\frac{1}{4}}\right)^4=(-6)^4$
$x=1296$
if $x^{\frac{1}{4}}=3:$
$(x^{\frac{1}{4}})^4=(3)^4$
$x=81$

Number patterns 4

Exercise 4 - 1

1. Write down the next three terms in each of the following sequences:
   (a) $5;\ 15;\ 25;\ \ldots$
   (b) $-8; -3;\ 2 : `\ldots$
   (c) $30;\ 27;\ 24;\ \ldots$
2. The general term is given for each sequence below. Calculate the missing terms.
   (a) $0; 3;\ \ldots;\ 15;\ 24$ $\qquad T_n = n^2 - 1$
   (b) $3;\ 2;\ 1;\ 0;\ \ldots;\ -2$ $\qquad T_n = -n + 4$
   (c) $-11;\ \ldots;\ -7;\ \ldots;\ -3$ $\qquad T_n = -13 + 2n$
3. Find the general formula for the following sequences and then find T_{10}, T_{50} and T_{100}
   (a) $2;\ 5;\ 8;\ 11;\ 14;\ \ldots$
   (b) $0;\ 4;\ 8;\ 12;\ 16;\ \ldots$
   (c) $2;\ -1;\ -4;\ -7;\ -10;\ \ldots$

Solutions to Exercise 4 - 1

1. (a) $35; 45; 55$
   (b) $7; 12; 7$
   (c) $21; 18; 15$
2. (a) $T_n = n^2 - 1$
   $T_3 = 3^2 - 1$
   $= 9 - 1$
   $= 8$
   (b) $T_n = -n + 4$
   $T_5 = -5 + 4$
   $= -1$
   (c) $T_n = -13 + 2n$
   $T_2 = -13 + 2(2)$
   $= -13 + 4$
   $= -9$
   $T_n = -13 + 2n$
   $T_4 = -13 + 2(4)$

$= -13 + 8$
$= -5$

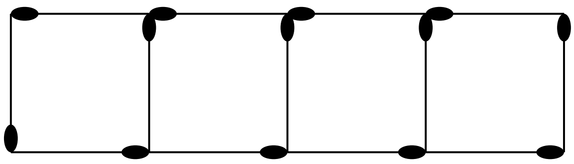
3. **(a)** $a = 2$
$d = 3$
$T_n = 3n - 1$
$T_{10} = 3(10) - 1 = 29$
$T_{50} = 3(50) - 1 = 149$
$T_{100} = 3(100) - 1 = 299$

(b) $a = 0$
$d = 4$
$T_n = 4n - 4$
$T_{10} = 4(10) - 4 = 36$
$T_{50} = 4(50) - 4 = 196$
$T_{100} = 4(100) - 4 = 396$

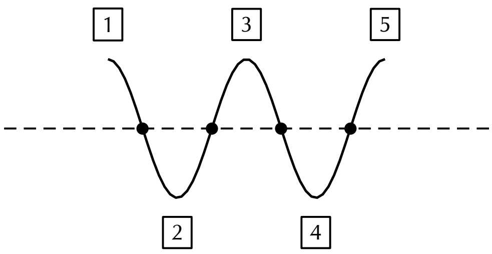
(c) $a = 2$
$d = -3$
$T_n = 5 - 3n$
$T_{10} = 5 - 3(10) = -25$
$T_{50} = 5 - 3(50) = -145$
$T_{100} = 5 - 3(100) = -295$

End of Chapter Exercises

1. Find the 6^{th} term in each of the following sequences:
   (a) $4;\ 13;\ 22;\ 31;\ \ldots$
   (b) $5;\ 2;\ -1;\ -4;\ \ldots$
   (c) $7{,}4;\ 9{,}7;\ 12;\ 14{,}3;\ \ldots$
2. Find the general term of the following sequences:
   (a) $3;\ 7;\ 11;\ 15;\ \ldots$
   (b) $-2;\ 1;\ 4;\ 7;\ \ldots$
   (c) $11;\ 15;\ 19;\ 23;\ \ldots$
   (d) $\frac{1}{3};\ \frac{2}{3};\ 1;\ 1\frac{1}{3};\ \ldots$
3. The seating of a sports stadium is arranged so that the first row has 15 seats, the second row has 19 seats, the third row has 23 seats and so on. Calculate how many seats are in the twenty-fifth row.
4. A single square is made from 4 matchsticks. Two squares in a row need 7 matchsticks and three squares in a row need 10 matchsticks. For this sequence determine:
   (a) the first term;
   (b) the common difference;
   (c) the general formula;
   (d) how many matchsticks there are in a row of twenty-five squares.



5. You would like to start saving some money, but because you have never tried to save money before, you decide to start slowly. At the end of the first week you deposit R 5 into your bank account. Then at the end of the second week you deposit R 10 and at the end of the third week, R 15. After how many weeks will you deposit R 50 into your bank account?
6. A horizontal line intersects a piece of string at 4 points and divides it into five parts, as shown below.



If the piece of string is intersected in this way by 19 parallel lines, each of which intersects it at 4 points, determine the number of parts into which the string will be divided.

7. Consider what happens when you add 9 to a two-digit number:

$$\begin{aligned} 9+16 &= 25 \\ 9+28 &= 37 \\ 9+43 &= 52 \end{aligned}$$

(a) What pattern do you see?
(b) Make a conjecture and express it in words.
(c) Generalise your conjecture algebraically.
(d) Prove that your conjecture is true.

Solutions to End of Chapter Exercises

1. **(a)** $T_6 = 49$
(b) $T_6 = -10$
(c) $T_6 = 18{,}9$

2. **(a)** $T_n = 4n - 1$
(b) $T_n = 3n - 5$
(c) $T_n = 4n + 7$
(d) $T_n = \frac{n}{3}$

3. Extract the relevant information which is a sequence: $15; 19; 23 \ldots$
   $a = 15$
   $d = 4$
   $T_n = 4n + 11$
   So the number of seats in row 25 is:
   $T_{25} = 4(25) + 11 = 111$

4. **(a)** The first term is 4 since this is the number of matches needed for 1 square.
   (b) The common difference between the terms is 3. $(7 - 4 = 3$; $10 - 7 = 3)$
   (c) $T_n = 3n + 1$
   (d) $T_25 = 3(25) + 1$
   $= 76$

5. Write down the sequence:
   $5; 10; 15; \ldots$ We first find the general formula:
   $a = 5$
   $d = 5$
   $T_n = 5n$
   $50 = 5n$
   $n = 10$ weeks

6. With one line intersecting at four points we get five parts. If we add a second line it is now broken up into 9 parts. And if we add a third line it is now broken up into 13 parts. So we see that for each line added we add four parts. The sequence is: $5; 9; 13 \ldots$
   $a = 5$
   $d = 4$
   $T_n = 4n + 1$
   $T_{19} = 4(19) + 1$
   $T_{19} = 77$

7. **(a)** The first digit of the answer increases by one number and the second digit decreases by one number.
   (b) When 9 is added to any two-digit number, the answer is the two-digit number, with its first ('tens') digit increased by one number and its second ('ones') digit decreased by one number.
   (c) $9 + (10x + y) = 10(x + 1) + (y - 1)$
   (d) $9 + 16 = 25$
   $9 + (10x + y) = 10(x + 1) + (y - 1)$
   $9 + (10 + 6) = 10(2) + (6 - 1)$
   $9 + 16 = 20 + 5 = 25$
   $9 + (10x + y) = 10x + 10 + y - 1$
   $9 + (10x + y) = (10 - 1) + (10x + y) = 9 + (10x + y)$

Functions 5

Exercise 5 - 1

1. Write the following in set notation:

   (a) $(-\infty; 7]$
   (b) $[13; 4)$
   (c) $(35; \infty)$
   (d) $[\frac{3}{4}; 21)$
   (e) $[-\frac{1}{2}; \frac{1}{2}]$
   (f) $(-\sqrt{3}; \infty)$

2. Write the following in interval notation:

   (a) $\{p : p \in \mathbb{R}, p \leq 6\}$
   (b) $\{k : k \in \mathbb{R}, \; -5 < k < 5\}$
   (c) $\{x : x \in \mathbb{R}, x > \frac{1}{5}\}$
   (d) $\{z : z \in \mathbb{R}, 21 \leq z < 41\}$

Solutions to Exercise 5 - 1

1. (a) $\{x : x \in \mathbb{R}, x \leq 7\}$
   (b) $\{y : y \in \mathbb{R}, -13 \leq y < 4\}$
   (c) $\{z : z \in \mathbb{R}, z > 35\}$
   (d) $\{t : t \in \mathbb{R}, \frac{3}{4} \leq t < 21\}$
   (e) $\{p : p \in \mathbb{R}, -\frac{1}{2} \leq p \leq \frac{1}{2}\}$
   (f) $\{m : m \in \mathbb{R}, m > -\sqrt{3}\}$

2. (a) $(-\infty; 6]$
   (b) $(-5; 5)$
   (c) $(\frac{1}{5}; \infty)$
   (d) $[21; 41)$

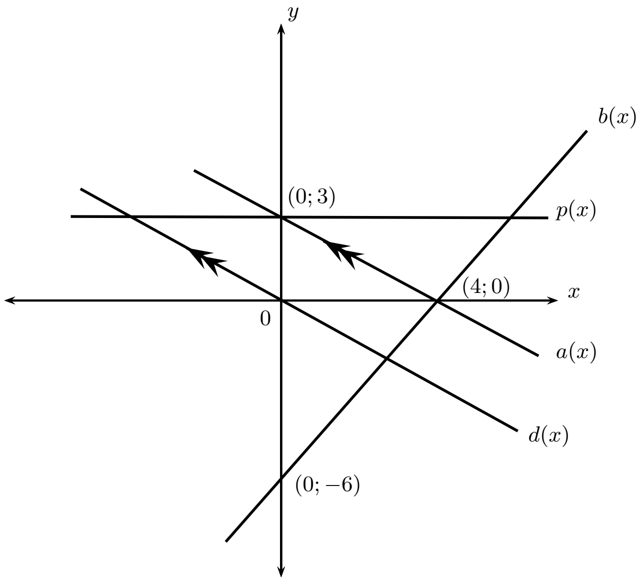
Exercise 5 - 2

1. List the x and y-intercepts for the following straight line graphs. Indicate whether the graph is increasing or decreasing:

(a) $y = x + 1$
(b) $y = x - 1$
(c) $h(x) = 2x - 1$
(d) $y + 3x = 1$
(e) $3y - 2x = 6$
(f) $k(x) = -3$
(g) $x = 3y$
(h) $\frac{x}{2} - \frac{y}{3} = 1$

2. For the functions in the diagram below, give the equation of the line:

(a) $a(x)$
(b) $b(x)$
(c) $p(x)$
(d) $d(x)$



3. Sketch the following functions on the same set of axes, using the dual intercept method. Clearly indicate the intercepts and the point of intersection of the two graphs: $x + 2y - 5 = 0$ and $3x - y - 1 = 0$

4. On the same set of axes, draw the graphs of $f(x) = 3 - 3x$ and $g(x) = \frac{1}{3}x + 1$ using the gradient–intercept method.

Solutions to Exercise 5 - 2

1. **(a)** $(0;1)$ and $(-1;0)$; increasing
(b) $(0;-1)$ and $(1;0)$; increasing
(c) $(0;-1)$ and $(\frac{1}{2};0)$; increasing
(d) $(0;1)$ and $(\frac{1}{3};0)$; decreasing
(e) $(0;2)$ and $(-3;0)$; increasing
(f) $(0;3)$; horizontal line
(g) $(0;0)$; increasing
(h) $(0;3)$ and $(2;0)$; decreasing

2.

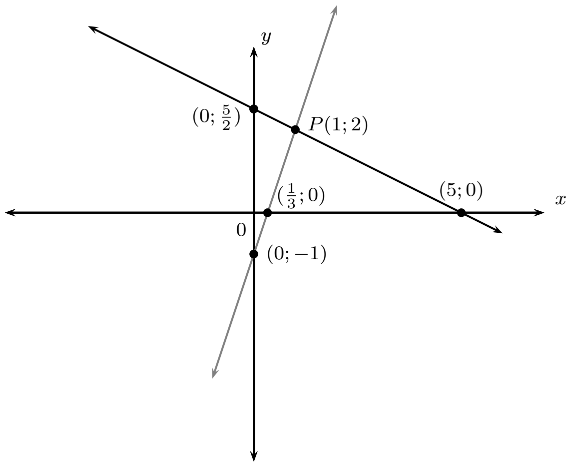
(a) $a(x) = -\frac{3}{4}x + 3$

(b) $b(x) = \frac{3}{2}x - 6$

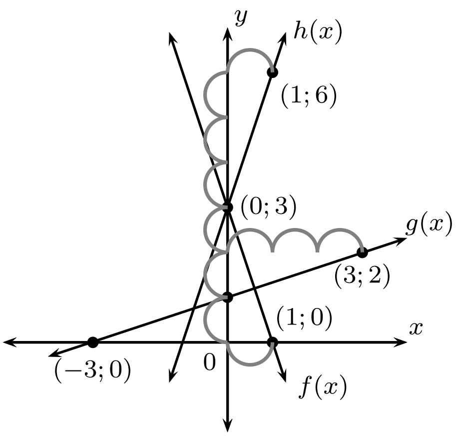
(c) $p(x) = 3$

(d) $d(x) = -\frac{3}{4}x$

3.

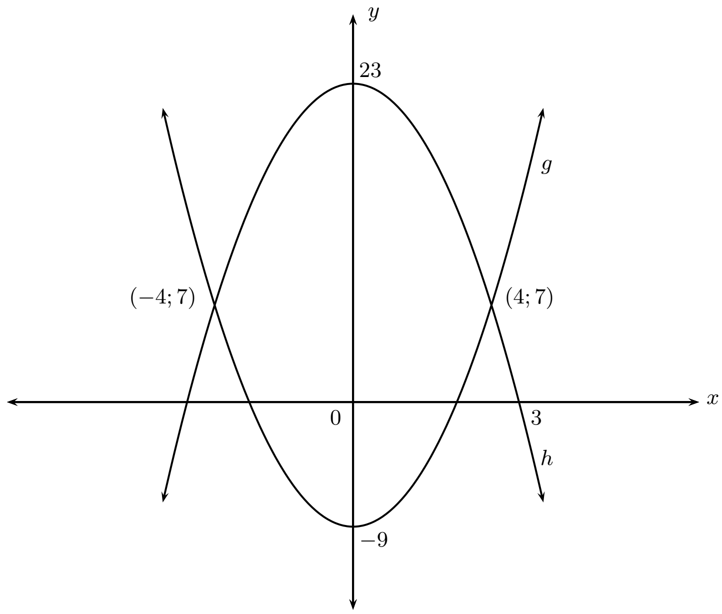


4.



Exercise 5 - 3

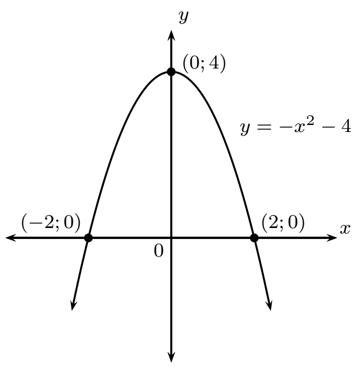
1. Show that if $a < 0$ the range of $f(x) = ax^2 + q$ is $\{f(x) : f(x) \leq q\}$.
2. Draw the graph of the function $y = -x^2 + 4$ showing all intercepts with the axes.
3. Two parabolas are drawn: $g : y = ax^2 + p$ and $h : y = bx^2 + q$.



(a) Find the values of a and p.
(b) Find the values of b and q.
(c) Find the values of x for which $g(x) \geq h(x)$.
(d) For what values of x is g increasing?

Solutions to Exercise 5 - 3

1. Because the square of any number is always positive we get: $x^2 \geq 0$
   If we multiply by a where $(a < 0)$ then the sign of the inequality is reversed: $ax^2 \leq 0$
   Adding q to both sides gives $ax^2 + q \leq q$
   And so $f(x) \leq q$
   This gives the range as $(-\infty; q)$.

2.
3. **(a)** p is the y-intercept, therefore $p = -9$
   To find a we use one of the points on the graph (e.g. $(4; 7)$):

$y = ax^2 - 9$
$7 = a(4^2) - 9$
$16a = 16$
$\therefore a = 1$

(b) q is the y-intercept, therefore $q = 23$
To find b, we use one of the points on the graph (e.g. $(4; 7)$):
$y = bx^2 = 23$
$7 = b(4^2) + 23$
$16b = -16$
$\therefore b = -1$

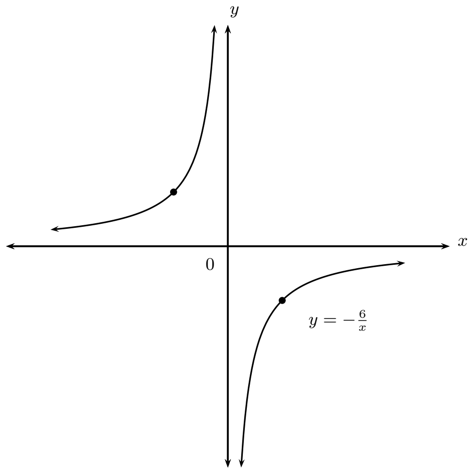
(c) This is the point where g lies above h. From the graph we see that g lies above h when: $x \leq -4$ or $x \geq 4$

(d) g increases from the turning point $(0; -9)$, i.e. for $x \geq 0$

Exercise 5 - 4

1. Draw the graph of $xy = -6$.
   (a) Does the point $(-2; 3)$ lie on the graph? Give a reason for your answer.
   (b) If the x-value of a point on the drawn graph is $0{,}25$ what is the corresponding y-value?
   (c) What happens to the y-values as the x-values become very large?
   (d) Give the equations of the asymptotes.
   (e) With the line $y = -x$ as line of symmetry, what is the point symmetrical to $(-2; 3)$?
2. Draw the graph of $h(x) = \frac{8}{x}$.
   (a) How would the graph $g(x) = \frac{8}{x} + 3$ compare with that of $h(x) = \frac{8}{x}$? Explain your answer fully.
   (b) Draw the graph of $y = \frac{8}{x} + 3$ on the same set of axes, showing asymptotes, axes of symmetry and the coordinates of one point on the graph.

Solutions to Exercise 5 - 4

1. 

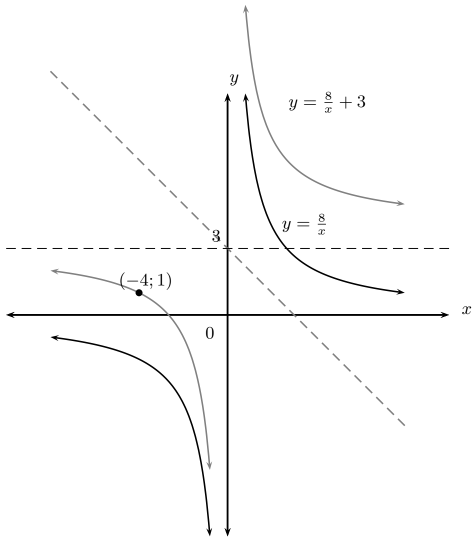
   (a) $y = \frac{-6}{x}$
   $xy = -6$
   Substitute the values of the point $(-2; 3)$ into the function:
   $xy = (-2)(3) = -6$
   This satistfies the equation therefore the point does lie on the graph.

   (b) Substitute in the value of x:
   $y = \frac{-6}{0{,}25}$
   $= -6 \times 4$
   $= -24$

   (c) The y-values decrease as the x-values become very large. The larger the denominator (x), the smaller the result of the fraction (y).

   (d) The graph is not vertically or horizontally shifted, therefore the asymptotes are $y = 0$ and $x = 0$.

   (e) Across the line of symmetry $y = -x$, the point symmetrical to $(-2; 3)$ is $(-3; 2)$.

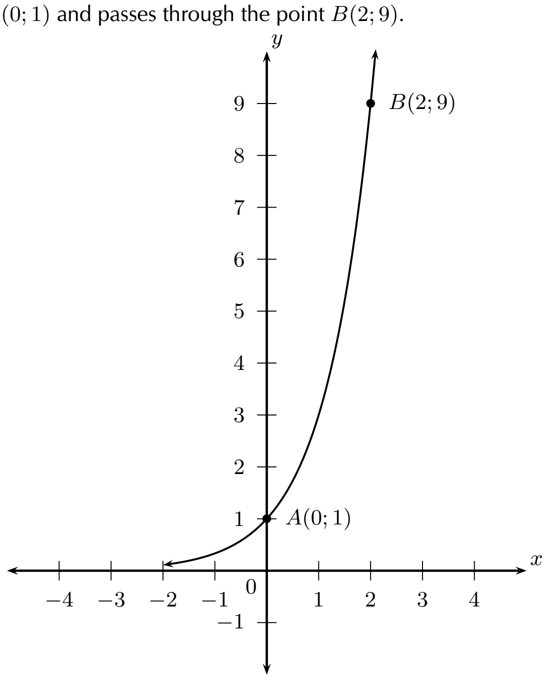
2. 

   (a) The graph $g(x) = \frac{8}{x} + 3$ is the graph of $h(x) = \frac{8}{x}$, vertically shifted upwards by 3 units. They would be the same shape but the asymptote of $g(x)$ would be $y = 3$, instead of $y = 0$ (for $h(x)$) and the axis of symmetry would be $y = -x + 3$ instead of $y = -x$ (for $h(x)$).

(b) Graph of $y = \frac{8}{x} + 3$ is on the set of axes above.

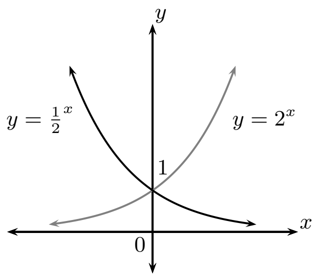
Exercise 5 - 5

1. Draw the graphs of $y = 2^x$ and $y = (\frac{1}{2})^x$ on the same set of axes.
   (a) Is the x-axis an asymptote or an axis of symmetry to both graphs? Explain your answer.
   (b) Which graph is represented by the equation $y = 2^{-x}$? Explain your answer.
   (c) Solve the equation $2^x = (\frac{1}{2})^x$ graphically and check that your answer is correct by using substitution.
2. The curve of the exponential function f in the accompanying diagram cuts the y-axis at the point $A(0; 1)$ and passes through the point $B(2; 9)$.



   (a) Determine the equation of the function f.
   (b) Determine the equation of h, the reflection of f in the x-axis.
   (c) Determine the range of h.
   (d) Determine the equation of g, the reflection of f in the y-axis.
   (e) Determine the equation of j if j is a vertical stretch of f by $+2$ units.
   (f) Determine the equation of k if k is a vertical shift of f by -3 units.

Solutions to Exercise 5 - 5

1. 

   (a) The x-axis is an asymptote to both graphs because both approach the x-axis but never touch it.

   (b) $y = \frac{1}{2}^x$ is represented by the equation $y = 2^{-x}$ because $\frac{1}{2} = 2^{-1}$, which gives $y = 2^{-x}$.

   (c) The graphs intersect at the point $(0; 1)$. If we substitute these values into each side of the equation we get:
   LHS: $2^x = 2^0 = 1$ and
   RHS: $\left(\frac{1}{2}\right)^x = \left(\frac{1}{2}\right)^0 = 1$
   LHS = RHS, therefore the answer is correct.

2. **(a)** The general form of the equation is $f(x) = a^x + q$.
   We are given $A(0; 1)$ and $B(2; 9)$.
   Substitute in the values of point A:
   $1 = a^0 + q$
   $1 = 1 + q$
   $\therefore q = 0$
   Substitute in the values of point B:
   $9 = a^2 + 0$
   $a^2 = 3^2$
   $\therefore a = 3$
   Therefore the equation is $f(x) = 3^x$.

   (b) $h(x) = -3^x$

   (c) Range: $(-\infty; 0)$

   (d) $g(x) = 3^{-x}$

   (e) $j(x) = 2.3^x$

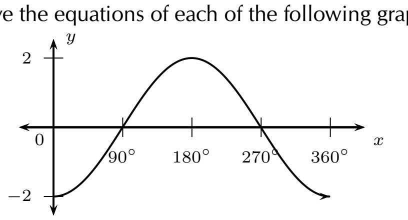
   (f) $k(x) = 3^x - 3$

Exercise 5 - 6

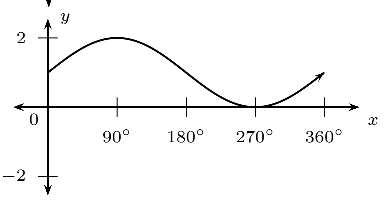
1. Using your knowledge of the effects of a and q, sketch each of the following graphs, without using a table of values, for $\theta \in [\,0°; 360°]$.

   (a) $y = 2\ sin\ \theta$
   (b) $y = -4\ cos\ \theta$
   (c) $y = -2\ cos\ \theta + 1$
   (d) $y = sin\ \theta - 3$
   (e) $y = tan\ \theta - 2$
   (f) $y = 2\ cos\ \theta - 1$

2. Give the equations of each of the following graphs:



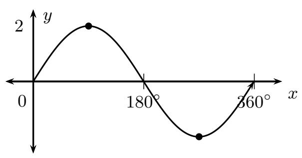
(a)



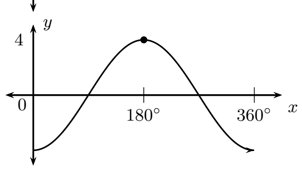
(b)

Solutions to Exercise 5 - 6

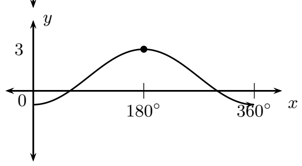
1. (a)



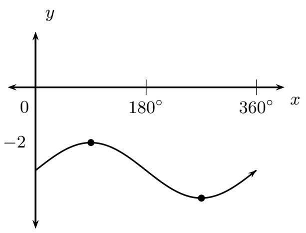
(b)



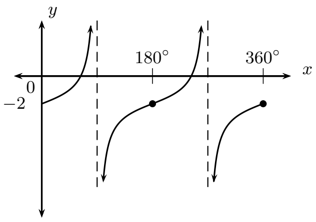
(c)

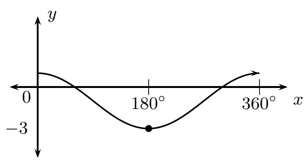


(d)



(e)





(f)

2. **(a)** $y = 2\ cos\ \theta$
   (b) $y = sin\ \theta + 1$

End of Chapter Exercises

1. Sketch the graphs of the following:

   (a) $y = 2x + 4$ **(b)** $y - 3x = 0$ **(c)** $2y = 4 - x$

2. Sketch the following functions:

   (a) $y = x^2 + 3$ **(b)** $y = \frac{1}{2}x^2 + 4$ **(c)** $y = 2x^2 - 4$

3. Sketch the following functions and identify the asymptotes:

   (a) $y = 3^x + 2$ **(b)** $y = -4 \times 2^x$ **(c)** $y = \left(\frac{1}{3}\right)^x - 2$

4. Sketch the following functions and identify the asymptotes:

   (a) $y = \frac{3}{x} + 4$ **(b)** $y = \frac{1}{x}$ **(c)** $y = \frac{2}{x} - 2$

5. Determine whether the following statements are true or false. If the statement is false, give reasons why:
   (a) The given or chosen y-value is known as the independent variable.
   (b) A graph is said to be congruent if there are no breaks in the graph.
   (c) Functions of the form $y = ax + q$ are straight lines.
   (d) Functions of the form $y = \frac{a}{x} + q$ are exponential functions.
   (e) An asymptote is a straight line which a graph will intersect at least once.
   (f) Given a function of the form $y = ax + q$, to find the y-intercept let $x = 0$ and solve for y.

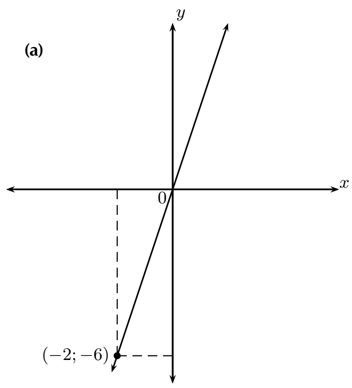
6. Given the functions $f(x) = 2x^2 - 6$ and $g(x) = -2x + 6$:
   (a) Draw f and g on the same set of axes.
   (b) Calculate the points of intersection of f and g.
   (c) Use your graphs and the points of intersection to solve for x when:
      i. $f(x) > 0$
      ii. $g(x) < 0$
      iii. $f(x) \leq g(x)$
   (d) Give the equation of the reflection of f in the x-axis.

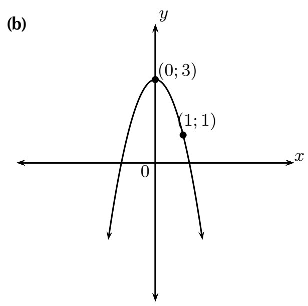
7. After a ball is dropped, the rebound height of each bounce decreases. The equation $y = 5(0{,}8)^x$ shows the relationship between the number of bounces x and the height of the bounce y for a certain ball. What is the approximate height of the fifth bounce of this ball to the nearest tenth of a unit?

8. Mark had 15 coins in R 5 and R 2 pieces. He had 3 more R 2 coins than R 5 coins. He wrote a system of equations to represent this situation, letting x represent the number of R 5 coins and y represent the number of R 2 coins. Then he solved the system by graphing.
   (a) Write down the system of equations.
   (b) Draw their graphs on the same set of axes.
   (c) Use your sketch to determine how many R 5 and R 2 pieces Mark had.

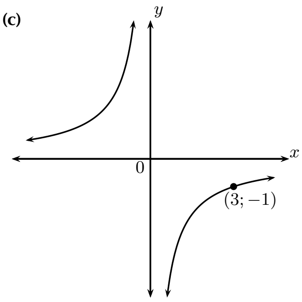
9. Sketch graphs of the following trigonometric functions for $\theta \in [\,0°; 360°]$. Show intercepts and asymptotes.
   (a) $y = -4\ cos\ \theta$
   (b) $y = sin\ \theta - 2$
   (c) $y = -2\ sin\ \theta + 1$
   (d) $y = tan\ \theta + 2$
   (e) $y = \frac{cos\ \theta}{2}$

10. Given the general equations $y = mx + c$, $y = ax^2 + q$, $y = \frac{a}{x} + q$, $y = a\ sin\ x + q$, $y = a^x + q$ and $y = a\ tan\ x$, determine the specific equations for each of the following graphs:

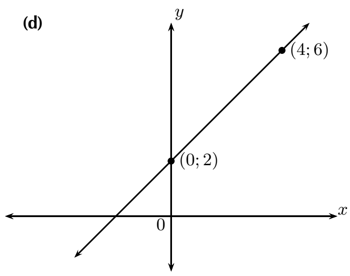




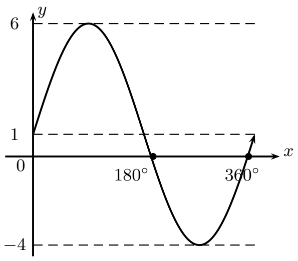
(c)



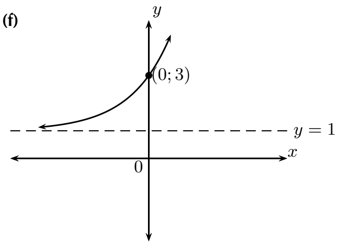
(d)



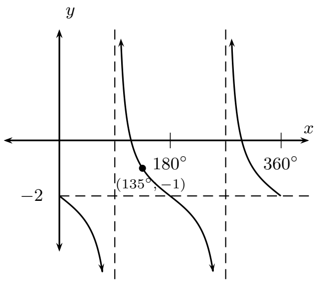
(e)



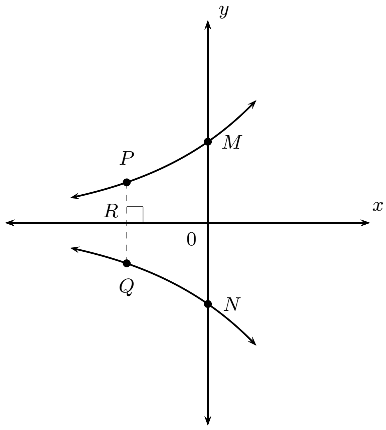
(f)



(g)



11. $y = 2^x$ and $y = -2^x$ are sketched below. Answer the questions that follow:



(a) Calculate the coordinates of M and N.

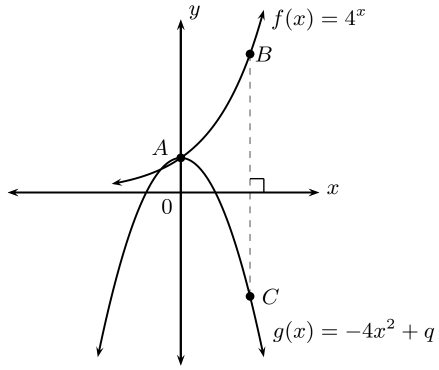
(b) Calculate the length of MN.

(c) Calculate length PQ if $OR = 1$ unit.

(d) Give the equation of $y = 2^x$ reflected about the y-axis.

(e) Give the range of both graphs.

12. $f(x) = 4^x$ and $g(x) = 4x^2 + q$ are sketched below. The points $A(0; 1)$, $B(1; 4)$ and C are given. Answer the questions that follow:



(a) Determine the value of q.

(b) Calculate the length of BC.

(c) Give the equation of $f(x)$ reflected about the x-axis.

(d) Give the equation of $f(x)$ shifted vertically upwards by 1 unit.

(e) Give the equation of the asymptote of $f(x)$.

(f) Give the ranges of $f(x)$ and $g(x)$.

13. Sketch the graphs $h(x) = x^2 - 4$ and $k(x) = -x^2 + 4$ on the same set of axes and answer the questions that follow:

(a) Describe the relationship between h and k.

(b) Give the equation of $k(x)$ reflected about the line $y = 4$.

(c) Give the domain and range of h.

14. Sketch the graphs $f(\theta) = 2sin\ \theta$ and $g(\theta) = cos\ \theta - 1$ on the same set of axes. Use your sketch to determine:

(a) $f(180°)$

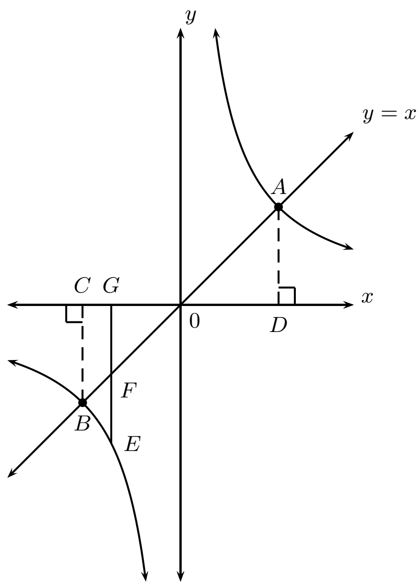
(b) $g(180°)$

(c) $g(270°) - f(270°)$

(d) The domain and range of g.

(e) The amplitude and period of f.

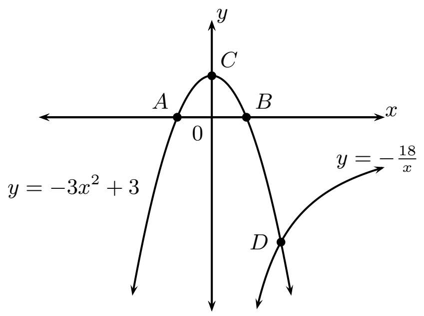
15. The graphs of $y = x$ and $y = \frac{8}{x}$ are shown in the following diagram.



Calculate:

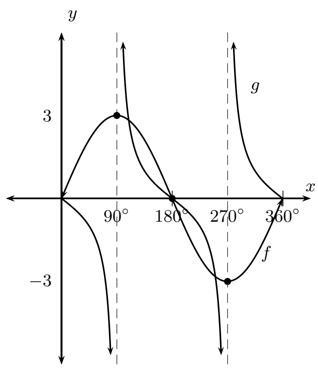
(a) the coordinates of points A and B.
(b) the length of CD.
(c) the length of AB.
(d) the length of EF, given $G(-2;0)$.

16. Given the diagram with $y = -3x^2 + 3$ and $y = -\frac{18}{x}$.



(a) Calculate the coordinates of A, B and C.
(b) Describe in words what happens at point D.
(c) Calculate the coordinates of D.
(d) Determine the equation of the straight line that would pass through points C and D .

17. The diagram shows the graphs of $f(\theta) = 3\ sin\ \theta$ and $g(\theta) = -tan\ \theta$.



(a) Give the domain of g.

(b) What is the amplitude of f?

(c) Determine for which values of θ:

i. $f(\theta) = 0 = g(\theta)$

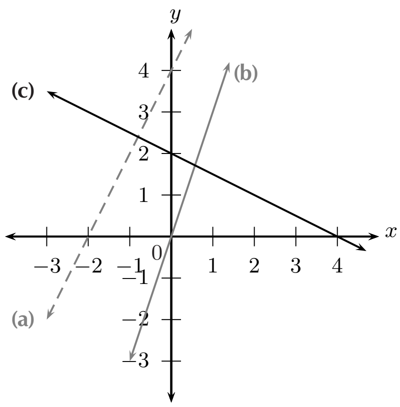
ii. $f(\theta) \times g(\theta) < 0$

iii. $\dfrac{g(\theta)}{f(\theta)} > 0$

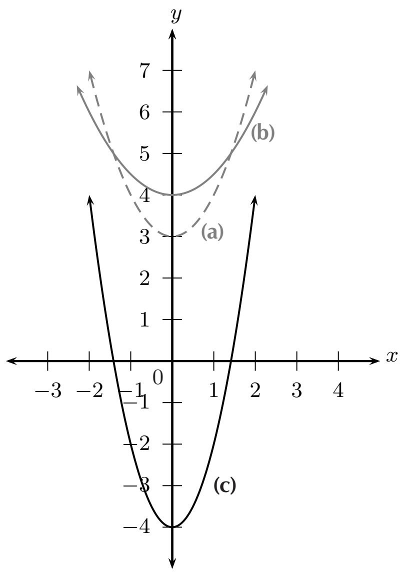
iv. $f(\theta)$ is increasing?

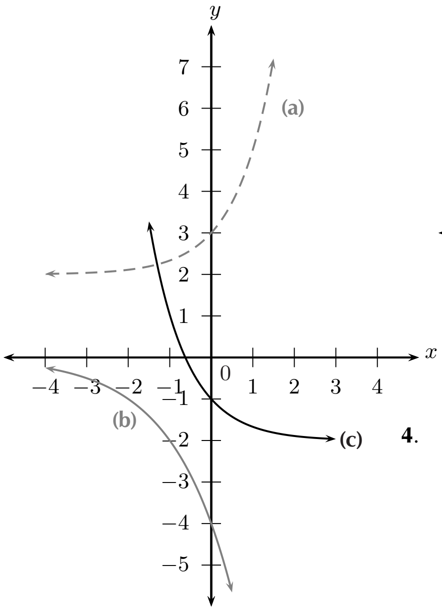
Solutions to End of Chapter Exercises

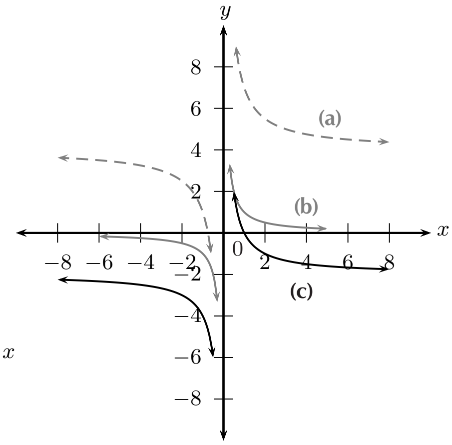
1.



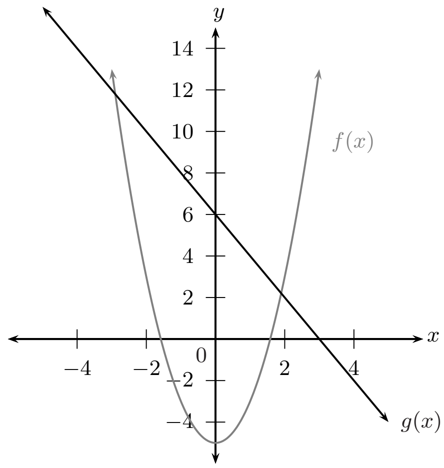
2.



3. 

4. 

5. **(a)** False - the given or chosen y-value is the dependent variable, because it's value depends on the independent variable x.
   (b) False - a graph is said to be *continuous* if there are no breaks in it.
   (c) True.
   (d) False - functions of the form $y = \frac{a}{x} + q$ are hyperbolic functions.
   (e) False - an asymptote is a straight line that a graph will *never* intersect.
   (f) True.

6. **(a)** 

   (b) The x-values of the points of intersection can be found by setting $f(x) = g(x)$:
   $2x^2 - 6 = -2x + 6$
   $2x^2 + 2x - 12 = 0$
   $x^2 + x - 6 = 0$
   $(x - 2)(x + 3) = 0$
   $\therefore x = 2$ and $x = -3$.
   The y-values can be obtained by substituting the x-values into either equation:
   $g(x) = -2(-3) + 6 = 12$

$g(x) = -2(2) + 6 = 2$

Therefore the points of intersection are $(-3;12)$ and $(2;2)$.

(c) **i.** Let $f(x) = 0$

$2x^2 - 6 = 0$

$2x^2 = 6$

$x^2 = 3$

$x = \pm\sqrt{3}$

Theferfore, for $f(x) > 0,\ x \in (-\infty;\sqrt{3}) \cup (\sqrt{3};\infty)$.

ii. Let $g(x) = 0$

$-2x + 6 = 0$

$-2x = -6$

$x = 3$

Therefore, for $g(x) < 0,\ x \in (3;\infty)$.

iii. For $f(x) \leq g(x),\ x \in [-3;2]$.

(d) $y = -2x^2 + 6$

7. $y = 5(0{,}8)^x$

$= 5(\frac{4}{5})^x$

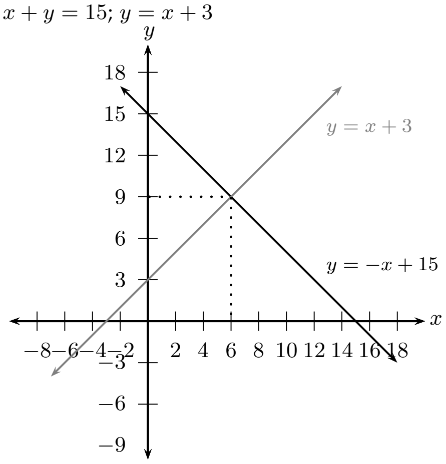
if $x = 5$ then $y = 5(\frac{4}{5})^5$

$= 5(\frac{1024}{3125})$

$= 5(0{,}38)$

$= 1{,}6$ units

8. **(a)** $x + y = 15$; $y = x + 3$



(b)

(c) Substitute value of $y = -x + 15$ into second equation:

$-x + 15 = x + 3$

$-2x = -12$

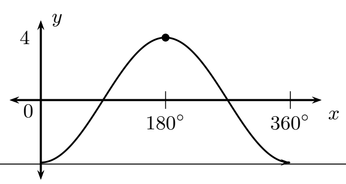
$\therefore x = 6$

Substitute value of x back into first equation:

$y = -(6) + 15$

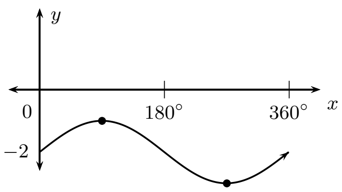
$= 9$

Mark has 6 R 5 coins and 9 R 2 coins.

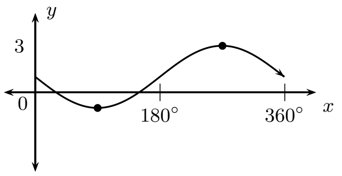


9. **(a)**

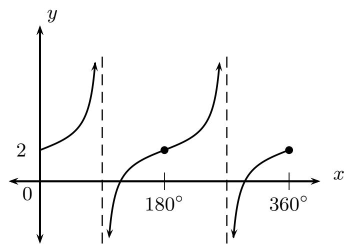
(b)



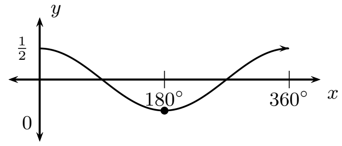
(c)



(d)



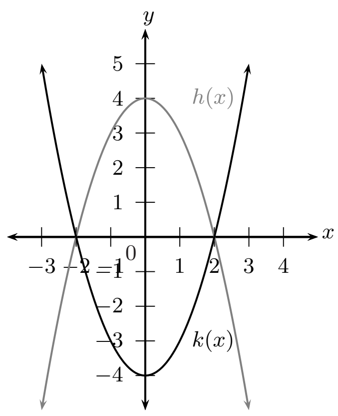
(e)



10. **(a)** $y = 3x$
(b) $y = -2x^2 + 3$
(c) $y = \frac{-3}{x}$
(d) $y = x + 2$
(e) $y = 5\ sin\ x + 1$
(f) $y = 2.2^x + 1$
(g) $y = -tan\ x - 2$

11. **(a)** At M, $y = 2^0 = 1$
therefore the coordinates of M are $(0; 1)$
At N, $y = -(2^0) = -1$
therefore the coordinates of N are$(0; -1)$
(b) $MN = 1 + 1 = 2$ units
(c) At P, $x = -1$
$\therefore y = 2^{-1} = \frac{1}{2}$.
At Q, $y = -(2^{-1}) = -\frac{1}{2}$.
Therefore length $PQ = \frac{1}{2} + \frac{1}{2} = 1$ unit.
(d) $y = 2^{-x}$
(e) Range $y = 2^x$: $(0; \infty)$
Range $y = -2^x$: $(-\infty; 0)$

12. **(a)** $q = 1$
(b) $BC = 7$ units
(c) $y = -4^x$
(d) $y = 4^x + 1$
(e) $x = 0$
(f) Range $f(x)$: $(0; \infty)$
Range $g(x)$: $(-\infty; 1]$



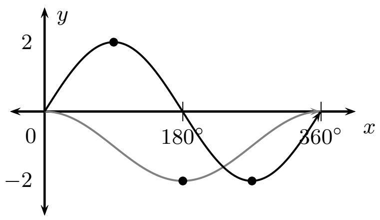
13.

(a) $h(x) = x^2 - 4$
$k(x) = -x^2 + 4$

$= -(x^2 - 4)$
$= -h(x)$
$k(x)$ is therefore the reflection of $h(x)$ about the x-axis.

(b) $y = x^2 + 4$

(c) Domain h: $(-\infty; \infty)$
Range h: $[-4; \infty)$

14. 

(a) $f(180°) = 0$

(b) $g(180°) = -2$

(c) $g(270°) - f(270°) = -1 - (-2)$
$= -1 + 2$
$= 1$

(d) Domain: $[0°; 360°]$
Range: $[-2; 0]$

(e) Amplitude $= 2$
Period $= 360°$

15. **(a)** $x = \frac{8}{x}$
$x^2 = 8$
$\therefore x = \pm\sqrt{8}$
Therefore $A(\sqrt{8}; \sqrt{8})$ and $B(-\sqrt{8}; -\sqrt{8})$

(b) $C(-\sqrt{8}; 0)$ and $D(\sqrt{8}; 0)$
$\therefore CD = \sqrt{8} + \sqrt{8} = 2\sqrt{8}$ units

(c) Using Pythagoras: $OD = \sqrt{8}$ units and $AD = \sqrt{8}$ units
$AO^2 = OD^2 + AD^2$
$= (\sqrt{8})^2 + (\sqrt{8})^2$
$= 8 + 8$
$= 16$
$\therefore AO = 4$ units
Similarly, $OB = 4$ units
$\therefore AB = 8$ units

(d) Given that $G(-2; 0)$, then $F(-2; -2)$ on the line $y = x$
Therefore length $EF = -2 - (-4) = 2$ units

16. **(a)** $y = -3x^2 + 3$
Let $y = 0$,
$0 = -3x^2 + 3$
$= x^2 - 1$
$= (x - 1)(x + 1)$
$\therefore x = -1$ and $x = 1$
Therefore $A(-1; 0)$ and $B(1; 0)$
When $x = 0, y = -3(0)^2 + 3$

Therefore $C(0;3)$.

(b) The parabola and the hyperbola intersect at point D which lies in the IV quadrant.

(c) $-\frac{18}{x} = -3x^2 + 3$
$-18 = -3x^3 + 3x$
$0 = -3x^3 + 3x + 18$
$0 = x^3 - x - 6$
$f(2) = (2)^3 - 2 - 6 = 0$
when $x = 2$, $y = -3(2)^2 + 3 = -9$
Therefore $D(2;-9)$.

(d) Determine gradient $D(2;-9)$ and $C(0;3)$:
$m = \frac{-9-3}{2-0} = -6$
Therefore $y = -6x + 3$

17. **(a)** Domain: $\{\theta : 0° \leq \theta \leq 360°, \theta \neq 90°; 270°\}$

(b) Amplitude $= 3$

(c) **i.** $\{0°;\ 180°;\ 360°\}$

ii. $(0°;\ 90°) \cup (270°;\ 360°)$

iii. $\{\theta : 90° < \theta < 270°, \theta \neq 180°\}$

iv. $(0°;\ 90°) \cup (270°;\ 360°)$

Finance and growth 6

Exercise 6 - 1

1. An amount of R $3\ 500$ is invested in a savings account which pays simple interest at a rate of $7{,}5\%$ per annum. Calculate the balance accumulated by the end of 2 years.
2. Calculate the accumulated amount in the following situations:
   (a) A loan of R 300 at a rate of 8% for 1 year.
   (b) An investment of R $2\ 250$ at a rate of $12{,}5\%$p.a. for 6 years.
3. Sally wanted to calculate the number of years she needed to invest R $1\ 000$ for in order to accumulate R $2\ 500$. She has been offered a simple interest rate of $8{,}2\%$ p.a. How many years will it take for the money to grow to R $2\ 500$?
4. Joseph made a deposit of R $5\ 000$ in the bank for his 5 year old son's 21^{st} birthday. He has given his son the amount of R $18\ 000$ on his birthday. At what rate was the money invested, if simple interest was calculated?

Solutions to Exercise 6 - 1

1. $P = 3\ 500$
   $i = 0{,}075$
   $n = 2$
   $A = ?$
   $A = P(1 + in)$
   $A = 3\ 500(1 + (0{,}075)(2))$
   $A = 3\ 500(1.15)$
   $A =$ R $4\ 025$

2. **(a)** $P = 300$
   $i = 0{,}08$
   $n = 1$
   $A = ?$
   $A = P(1 + in)$
   $A = 300(1 + (0{,}08)(1))$
   $A = 300(1{,}08)$
   $A =$ R 324

   (b) $P = 2\ 250$
   $i = 0{,}125$
   $n = 6$
   $A = ?$
   $A = P(1 + in)$
   $A = 2\ 250(1 + (0{,}125)(6))$
   $A = 2\ 250(1.75)$
   $A =$ R $3\ 937{,}50$

3. $A = 2\ 500$
   $P = 1\ 000$
   $i = 0{,}082$
   $n = ?$
   $A = P(1 + in)$
   $2\ 500 = 1\ 000(1 + (0{,}082)(n))$
   $\frac{2\ 500}{1\ 000} = 1 + 0{,}082n$
   $\frac{2\ 500}{1\ 000} - 1 = 0{,}082n$
   $(\frac{2\ 500}{1\ 000} - 1),082 = n$
   $n = 18{,}3$
   It would take 19 years for R $1\ 000$ to become R $2\ 500$ at $8{,}2\%$ p.a.

4. $A = 18\ 000$
   $P = 5\ 000$
   $i = ?$
   $n = 21 - 5 = 16$
   $A = P(1 + in)$
   $18\ 000 = 5\ 000(1 + (i)(16))$
   $\frac{18\ 000}{5\ 000} = 1 + 16i$
   $\frac{18\ 000}{5\ 000} - 1 = 16i$
   $(\frac{18\ 000}{5\ 000} - 1)6 = i$
   $i = 0{,}0125$
   The interest rate at which the money was invested was $1{,}25\%$.

Exercise 6 - 2

1. An amount of R $3\ 500$ is invested in a savings account which pays a compound interest rate of $7{,}5\%$ p.a. Calculate the balance accumulated by the end of 2 years.
2. Morgan invests R $5\ 000$ into an account which pays out a lump sum at the end of 5 years. If he gets R $7\ 500$ at the end of the period, what compound interest rate did the bank offer him?
3. Nicola wants to invest some money at a compound interest rate of 11% p.a. How much money (to the nearest Rand) should be invested if she wants to reach a sum of R $100\ 000$ in five years time?

Solutions to Exercise 6 - 2

1. $P = 3\ 500$
   $i = 0{,}075$
   $n = 2$
   $A = P(1+i)^n$
   $A = 3\ 500(1+0{,}075)^2$
   $A = \text{R } 4\ 044{,}69$

2. $A = 7\ 500$
   $P = 5\ 000$
   $i = ?$
   $n = 5$
   $A = P(1+i)^n$
   $7\ 500 = 5\ 000(1+i)^5$
   $\frac{7\ 500}{5\ 500} = (1+i)^5$
   $\sqrt[5]{\frac{7\ 500}{5\ 500}} = (1+i)$
   $\sqrt[5]{\frac{7\ 500}{5\ 500}} - 1 = i$
   $i = 0{,}0844717712$
   interest rate is $8{,}45\%$ p.a

3. $A = 100\ 000$
   $P = ?$
   $i = 0{,}11$
   $n = 5$
   $A = P(1+i)^n$
   $100\ 000 = P(1+0{,}11)^5$
   $\frac{100\ 000}{(1{,}11)^5} = P$
   $P = \text{R } 59\ 345{,}13$

Exercise 6 - 3

1. Vanessa wants to buy a fridge on a hire purchase agreement. The cash price of the fridge is R $4\ 500$. She is required to pay a deposit of 15% and pay the remaining loan amount off over 24 months at an interest rate of 12% p.a.
   (a) What is the principal loan amount?
   (b) What is the accumulated loan amount?

(c) What are Vanessa's monthly repayments?
(d) What is the total amount she has paid for the fridge?

2. Bongani buys a dining room table costing R 8 500 on a hire purchase agreement. He is charged an interest rate of 17,5% p.a. over 3 years.
   (a) How much will Bongani pay in total?
   (b) How much interest does he pay?
   (c) What is his monthly instalment?
3. A lounge suite is advertised on TV, to be paid off over 36 months at R 150 per month.
   (a) Assuming that no deposit is needed, how much will the buyer pay for the lounge suite once it has been paid off?
   (b) If the interest rate is 9% p.a., what is the cash price of the suite?

Solutions to Exercise 6 - 3

1. (a) $P = 4\ 500 - (4\ 500 \times 0{,}15)$
   $= 4\ 500 - 675$
   $=$ R $3\ 825$

   (b) $P =$ R $3\ 825$
   $i = 0{,}12$
   $n = \frac{24}{12} = 2$
   $A = P(1 + in)$
   $A = 3\ 825(1 + (0{,}12)(2))$
   $A =$ R $4\ 743$

   (c) $\dfrac{4\ 743}{24} =$ R $197{,}63$

   (d) $675 + 4743 =$ R $5\ 418$

2. (a) $A = ?$
   $P = 8\ 500$
   $i = 0{,}175$
   $n = 3$
   $A = P(1 + in)$
   $A = 8\ 500(1 + (0{,}175)(3))$
   $A =$ R $12\ 962{,}50$

   (b) $12\ 962{,}50 - 8\ 500 =$ R $4\ 462{,}50$

   (c) $\dfrac{12\ 962{,}50}{36} =$ R $360{,}07$

3. (a) $36 \times 150 =$ R $5\ 400$

   (b) $A = 5\ 400$
   $P = ?$
   $i = 0{,}09$
   $n = 3$
   $A = P(1 + in)$

$$5\ 400 = P(1 + (0{,}09)(3))$$
$$\frac{5\ 400}{1{,}27} = P$$
$$P = \text{R } 4\ 251{,}97$$

Exercise 6 - 4

1. If the average rate of inflation for the past few years was $7{,}3\%$ p.a. and your water and electricity account is R $1\ 425$ on average, what would you expect to pay in 6 years time?
2. The price of popcorn and a coke at the movies is now R 60. If the average rate of inflation is $9{,}2\%$ p.a. What was the price of popcorn and coke 5 years ago?
3. A small town in Ohio, USA is experiencing a huge increase in births. If the average growth rate of the population is 16% p.a., how many babies will be born to the $1\ 600$ residents in the next 2 years?

Solutions to Exercise 6 - 4

1. $A = ?$
   $P = 1\ 425$
   $i = 0{,}073$
   $n = 6$
   $A = P(1+i)^n$
   $A = 1\ 425(1 + 0{,}073)^6$
   $A = \text{R } 2\ 174{,}77$
2. $A = \text{R } 60$
   $P = ?$
   $i = 0{,}092$
   $n = 5$
   $A = P(1+i)^n$
   $60 = P(1 + 0{,}092)^5$
   $\frac{60}{(1{,}092)^5} = P$
   $P = \text{R } 38{,}64$
3. $A = ?$
   $P = 1\ 600$
   $i = 0{,}16$
   $n = 2$
   $A = P(1+i)^n$
   $A = 1\ 600(1 + 0{,}16)^2$

$A = 2\ 152{,}96$
$2\ 153 - 1\ 600 = 553$
There will be roughly 553 babies born in the next two years.

Exercise 6 - 5

1. Bridget wants to buy an iPod that costs £ 100, with the exchange rate currently at £ $1 =$ R 14. She estimates that the exchange rate will drop to R 12 in a month.
   (a) How much will the iPod cost in Rands, if she buys it now?
   (b) How much will she save if the exchange rate drops to R 12?
   (c) How much will she lose if the exchange rate moves to R 15?
2. Study the following exchange rate table:

Country	**Currency**	**Exchange Rate**
United Kingdom (UK)	Pounds (£)	R $14{,}13$
United States (USA)	Dollars ($)	R $7{,}04$

   (a) In South Africa the cost of a new Honda Civic is R $173\ 400$. In England the same vehicle costs £ $12\ 200$ and in the USA \$ $21\ 900$. In which country is the car the cheapest?
   (b) Sollie and Arinda are waiters in a South African restaurant attracting many tourists from abroad. Sollie gets a £ 6 tip from a tourist and Arinda gets \$ 12. Who got the better tip?

Solutions to Exercise 6 - 5

1. **(a)** Cost in Rands = (cost in Pounds) times exchange rate
   Cost in Rands $= 100 \times \frac{14}{1} =$ R $1\ 400$
   (b) Cost in Rands $= 100 \times \frac{12}{1} =$ R $1\ 200$
   So she will save R 200 (Saving $=$ R $1\ 400 -$ R $1\ 200$)
   (c) Cost in rands $= 100 \times \frac{15}{1} =$ R $1\ 500$
   So she will lose R 100 (Loss $=$ R $1\ 400 -$ R $1\ 500$)
2. **(a)** To answer this question we work out the cost of the car in rand for each country and then compare the three answers to see which is the cheapest. Cost in Rands $=$ cost in currency times exchange rate.
   Cost in UK: $12\ 200 \times \frac{14{,}13}{1} =$ R $172\ 386$
   Cost in USA: $21\ 900 \times \frac{7{,}04}{1} =$ R $154\ 400$
   Comparing the three costs we find that the car is the cheapest in the USA.

(b) Sollie: $6 \times \frac{14{,}31}{1} = \text{R } 84{,}78$
Arinda: $12 \times \frac{7{,}04}{1} = \text{R } 84{,}48$.

End of Chapter Exercises

1. Alison is going on holiday to Europe. Her hotel will cost € 200 per night. How much will she need in Rands to cover her hotel bill, if the exchange rate is € 1 = R $9{,}20$?
2. Calculate how much you will earn if you invested R 500 for 1 year at the following interest rates:
   (a) $6{,}85\%$ simple interest
   (b) $4{,}00\%$ compound interest
3. Bianca has R $1\ 450$ to invest for 3 years. Bank A offers a savings account which pays simple interest at a rate of 11% per annum, whereas Bank B offers a savings account paying compound interest at a rate of $10{,}5\%$ per annum. Which account would leave Bianca with the highest accumulated balance at the end of the 3 year period?
4. How much simple interest is payable on a loan of R $2\ 000$ for a year, if the interest rate is 10% p.a.?
5. How much compound interest is payable on a loan of R $2\ 000$ for a year, if the interest rate is 10% p.a.?
6. Discuss:
   (a) Which type of interest would you like to use if you are the borrower?
   (b) Which type of interest would you like to use if you were the banker?
7. Calculate the compound interest for the following problems.
   (a) A R $2\ 000$ loan for 2 years at 5% p.a.
   (b) A R $1\ 500$ investment for 3 years at 6% p.a.
   (c) A R 800 loan for 1 year at 16% p.a.
8. If the exchange rate to the Rand for Japanese Yen is ¥ 100 = R $6{,}2287$ and for Australian Dollar is 1 AUD = R $5{,}1094$, determine the exchange rate between the Australian Dollar and the Japanese Yen.
9. Bonnie bought a stove for R $3\ 750$. After 3 years she had finished paying for it and the R $956{,}25$ interest that was charged for hire purchase. Determine the rate of simple interest that was charged.
10. According to the latest census, South Africa currently has a population of $57\ 000\ 000$.
   (a) If the annual growth rate is expected to be $0{,}9\%$, calculate how many South Africans there will be in 10 years time (correct to the nearest hundred thousand).
   (b) If it is found after 10 years that the population has actually increased by 10 million to 67 million, what was the growth rate?

Solutions to End of Chapter Exercises

1. Cost in Rands = cost in Euros × exchange rate
   $= 200 \times 9{,}201$
   $=$ R $1\ 840$

2. (a) $P = 500$
   $i = 0{,}685$
   $n = 1$
   $A = ?$
   $A = P(1 + in)$
   $A = 500(1 + (0{,}685)(1))$
   $A = 500(1{,}685)$
   $A =$ R $534{,}25$

   (b) $P = 500$
   $i = 0{,}04$
   $n = 1$
   $A = ?$
   $A = P(1 + i)^n$
   $A = 500(1 + 0{,}04)^1$
   $A =$ R 520

3. Bank A:
   $P = 1\ 450$
   $i = 0{,}11$
   $n = 3$
   $A = ?$
   $A = P(1 + in)$
   $A = 1\ 450(1 + (0{,}11)(3))$
   $A = 1\ 450(1{,}33)$
   $A =$ R $1\ 925{,}50$
   Bank B:
   $P = 1\ 450$
   $i = 0{,}150$
   $n = 3$
   $A = ?$
   $A = P(1 + i)^n$
   $A = 1\ 450(1 + 0{,}150)^3$
   $A =$ R $1956{,}39$
   She should choose Bank B as it will give her more money after 3 years.

4. $P = 2\ 000$
   $i = 0{,}10$
   $n = 1$
   $A = ?$
   $A = P(1 + in)$
   $A = 2\ 000(1 + (0{,}10)(1))$
   $A = 2\ 000(1{,}10)$
   $A =$ R $2\ 200$
   So the amount of interest is:
   $2\ 200 - 2\ 000 =$ R 200

5. $P = 2\ 000$
   $i = 0{,}10$
   $n = 1$
   $A = ?$
   $A = P(1 + i)^n$
   $A = 2\ 000(1 + 0{,}10)^1$
   $A =$ R $2\ 200$

So the amount of interest is:
$2\ 200 - 2\ 000 =$ R 200

6. (a) Simple interest. Interest is only calculated on the principal amount and not on the interest earned during prior periods. This will lead to the borrower paying less interest.
   (b) Compound interest. Interest is calculated from the principal amount as well as interest earned from prior periods. This will lead to the banker getting more money for the bank.

7. (a) $P = 2\ 000$
   $i = 0{,}05$
   $n = 2$
   $A = ?$
   $A = P(1+i)^n$
   $A = 2\ 000(1 + 0{,}05)^2$
   $A =$ R $2\ 205$
   So the amount of interest is:
   $2\ 205 - 2\ 000 =$ R 205
   (b) $P = 1\ 500$
   $i = 0{,}06$
   $n = 3$
   $A = ?$
   $A = P(1+i)^n$
   $A = 1\ 500(1 + 0{,}06)^3$
   $A =$ R $1\ 786{,}524$
   So the amount of interest is:
   $1\ 786{,}524 - 1\ 500 =$ R $286{,}52$
   (c) $P = 800$
   $i = 0{,}16$
   $n = 1$
   $A = ?$
   $A = P(1+i)^n$
   $A = 800(1 + 0{,}16)^1$
   $A =$ R 928
   So the amount of interest is:
   $928 - 800 =$ R 128

8. $\frac{\text{AUD}}{\text{Yen}} = \frac{\text{ZAR}}{\text{Yen}} \times \frac{\text{AUD}}{\text{Yen}}$
   $= \frac{6{,}2287}{100} \times 15{,}1094$
   $= \frac{0{,}01219}{0{,}00219}$ AUD
   $= 1$ Yen
   or 1 AUD $= 82{,}03$ Yen

9. Total paid $= 3\ 750 + 956{,}25 = 4\ 706{,}25$
   $P = 3\ 750$
   $i = ?$
   $n = 3$
   $A = 4\ 706{,}25$

$A = P(1 + in)$
$4\ 706{,}25 = 3\ 750(1 + i(3))$
$1{,}255 = (1 + 3i)$
$0{,}255 = 3i$
$i = 0{,}085$
So the interest rate is $8{,}5\%$

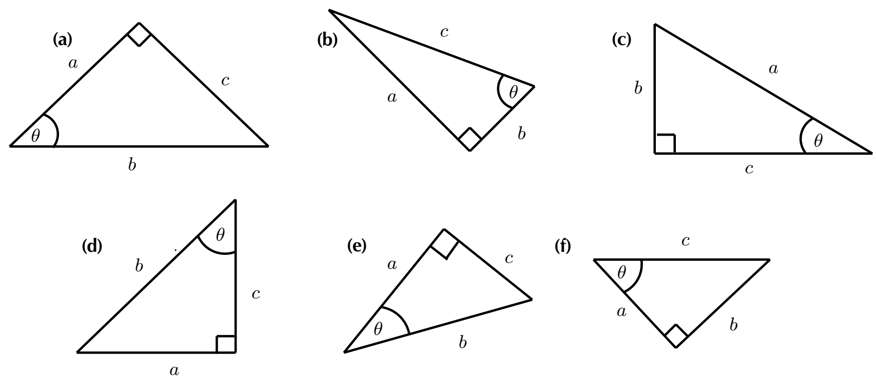
10. (a) $A = 57\ 000\ 000(1 + \frac{0{,}9}{100})^{10}$
$= 57\ 000\ 000(1{,}009)^{10}$
$= 57\ 000\ 000(1{,}0937)^{10}$
$=$ 62,3 million people

(b) $67 = 57(1 + \frac{i}{100})^{10}$
$\sqrt[10]{\frac{67}{57}} = 1 + \frac{i}{100}$
$\frac{i}{100} = 1{,}01629 - 1$
$i = 100(0{,}016)$
$i = 1{,}69$
$i \approx 1{,}7\%$

Trigonometry 7

Exercise 7 - 1

1. In each of the following triangles, state whether a, b and c are the hypotenuse, opposite or adjacent sides of the triangle with respect to θ.



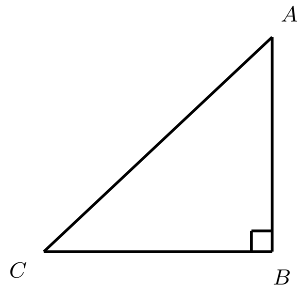
2. Use your calculator to determine the value of the following (correct to 2 decimal places):

(a) $tan\ 65°$
(b) $sin\ 38°$
(c) $cos\ 74°$
(d) $sin\ 12°$
(e) $cos\ 26°$
(f) $tan\ 49°$
(g) $\frac{1}{4}\ cos\ 20°$
(h) $3\ tan\ 40°$
(i) $\frac{2}{3}\ sin\ 90°$

3. If $x = 39°$ and $y = 21°$, use a calculator to determine whether the following statements are true or false:

(a) $cos\ x + 2\ cos\ x = 3\ cos\ x$
(b) $cos\ 2y = cos\ y + cos\ y$
(c) $tan\ x = \dfrac{sin\ x}{cos\ x}$
(d) $cos\ (x + y) = cos\ x + cos\ y$

4. Complete each of the following (the first example has been done for you):



(a) $sin\ \hat{A} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{CB}{AC}$

(b) $cos\ \hat{A} =$

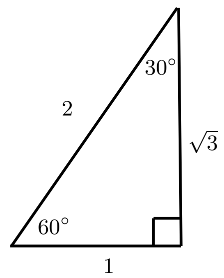
(c) $tan\ \hat{A} =$

(d) $sin\ \hat{C} =$

(e) $cos\ \hat{C} =$

(f) $tan\ \hat{C} =$

5. Use the triangle below to complete the following:



(a) $sin\ 60° =$

(b) $cos\ 60° =$

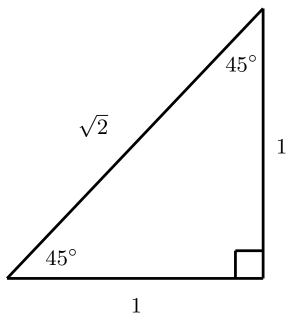
(c) $tan\ 60° =$

(d) $sin\ 30° =$

(e) $cos\ 30° =$

(f) $tan\ 30° =$

6. Use the triangle below to complete the following:



(a) $sin\ 45° =$

(b) $cos\ 45° =$

(c) $tan\ 45° =$

Solutions to Exercise 7 - 1

1. **(a)** a =adj; b =hyp; c =opp
   (b) a =opp; b =adj; c =hyp
   (c) a =hyp; b =opp; c =adj
   (d) a =opp; b =hyp; c =adj
   (e) a =adj; b =hyp; c =opp
   (f) a =adj; b =opp; c =hyp

2. **(a)** $2{,}14$
   (b) $0{,}62$
   (c) $0{,}28$
   (d) $0{,}21$
   (e) $0{,}90$
   (f) $1{,}15$
   (g) $0{,}23$
   (h) $2{,}52$
   (i) $0{,}67$

3. **(a)** LHS $= cos\ 39° + 2\ cos\ 39°$
   $= 2{,}33$
   RHS $= 3\ cos\ 39°$
   $= 2{,}33$
   LHS=RHS, therefore statement is true.
   (b) LHS $= cos\ 2(21°)$
   $= 0{,}74$
   RHS $= cos\ 21° + cos\ 21°$
   $= 1{,}87$
   LHS $\neq$ RHS, therefore statement is false.
   (c) LHS $= tan\ 39°$
   $= 0{,}81$
   RHS $= \frac{sin\ 39°}{cos\ 39°}$
   $= 0{,}81$
   LHS=RHS, therefore statement is true.
   (d) LHS $= cos\ (39° + 21°)$
   $= 0{,}5$
   RHS $= cos\ 39° + cos\ 21°$
   $= 1{,}71$
   LHS $\neq$ RHS, therefore statement is false.

4. **(a)** $sin\ \hat{A} = \frac{CB}{AC}$
   (b) $cos\ \hat{A} = \frac{AB}{AC}$
   (c) $tan\ \hat{A} = \frac{CB}{AB}$
   (d) $sin\ \hat{C} = \frac{AB}{AC}$
   (e) $cos\ \hat{C} = \frac{CB}{AC}$
   (f) $tan\ \hat{C} = \frac{AB}{CB}$

5. **(a)** $\frac{\sqrt{3}}{2}$
   (b) $\frac{1}{2}$
   (c) $\sqrt{3}$
   (d) $\frac{1}{2}$
   (e) $\frac{\sqrt{3}}{2}$
   (f) $\frac{1}{\sqrt{3}}$

6.

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{1}{\sqrt{2}}$
(c) 1

Exercise 7 - 2

1. Calculate the value of the following without using a calculator:
   (a) $sin\ 45° \times cos\ 45°$
   (b) $cos\ 60° + tan\ 45°$
   (c) $sin\ 60° - cos\ 60°$
2. Use the table to show that:
   (a) $\dfrac{sin\ 60°}{cos\ 60°} = tan\ 60°$
   (b) $sin^2 45° + cos^2 45° = 1$
   (c) $cos\ 30° = \sqrt{1 - sin^2 30°}$
3. Use the definitions of the trigonometric rations to answer the following questions:
   (a) Explain why $sin\ \alpha \leq 1$ for all values of α.
   (b) Explain why $cos\ \beta$ has a maximum value of 1.
   (c) Is there a maximum value for $tan\ \gamma$?

Solutions to Exercise 7 - 2

1. **(a)** $sin\ 45° \times cos\ 45°$
   $= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$
   $= \frac{1}{2}$

   (b) $cos\ 60° + tan\ 45°$
   $= \frac{1}{2} + 1$
   $= 1\frac{1}{2}$

   (c) $sin\ 60° - cos\ 60°$
   $= \frac{\sqrt{3}}{2} - \frac{1}{2}$
   $= \frac{\sqrt{3}-1}{2}$
2. **(a)** LHS$= \dfrac{sin\ 60°}{cos\ 60°}$
   $= \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$
   $= \frac{\sqrt{3}}{2} \times \frac{2}{1}$
   $= \sqrt{3}$

RHS $= tan\ 60°$
$= \sqrt{3}$
Therefore LHS=RHS.

(b) LHS= $sin^2\ 45° + cos^2\ 45°$
$= (\frac{1}{\sqrt{2}})^2 + (\frac{1}{\sqrt{2}})^2$
$= \frac{1}{2} + \frac{1}{2}$
$= 1$
RHS= 1
Therefore LHS=RHS.

(c) LHS= $cos\ 30°$
$= \frac{\sqrt{3}}{2}$
RHS= $\sqrt{1 - sin^2\ 30°}$
$= \sqrt{1 - (\frac{1}{2})^2}$
$= \sqrt{1 - (\frac{1}{4})}$
$= \sqrt{\frac{3}{4}}$
$= \frac{\sqrt{3}}{2}$
Therefore LHS=RHS.

3. **(a)** The sine ratio is defined as $\frac{\text{opposite}}{\text{hypotenuse}}$. In any right-angled triangle, the hypotenuse is the side of longest length. Therefore the maximum length of the opposite side is equal to the length of the hypotenuse. The maximum value of the sine ratio is then $\frac{\text{hypotenuse}}{\text{hypotenuse}} = 1$.

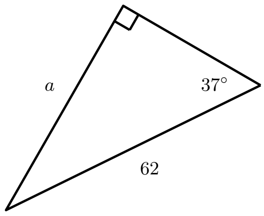
(b) Similarly, the cosine ratio is defined as $\frac{\text{adjacent}}{\text{hypotenuse}}$. In any right-angled triangle, the hypotenuse is the side of longest length. Therefore the maximum length of the adjacent side is equal to the length of the hypotenuse. The maximum value of the cosine ratio is then $\frac{\text{hypotenuse}}{\text{hypotenuse}} = 1$.

(c) For the tangent ratio $\frac{\text{opposite}}{\text{adjacent}}$, there is no maximum value. Notice that there is a restriction; the ratio is undefined if the length of the adjacent side equals 0.

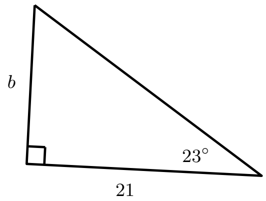
Exercise 7 - 3

1. In each triangle find the length of the side marked with a letter. Give answers correct to 2 decimal places.

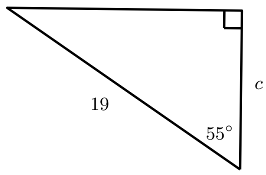
(a)



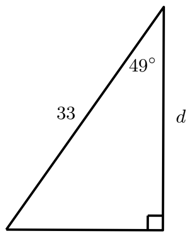
(b)



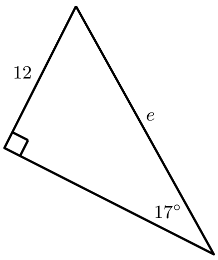
(c)



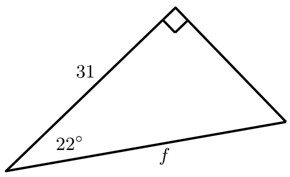
(d)



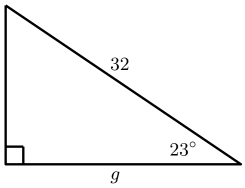
(e)



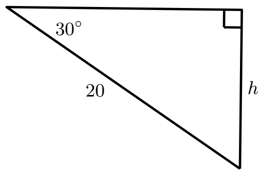
(f)



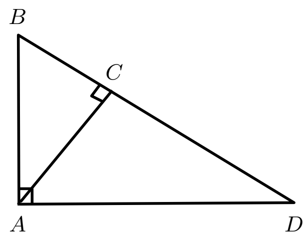
(g)



(h)



2. Write down two ratios for each of the following in terms of the sides: AB; BC; BD; AD; DC and AC:



(a) $sin\ \hat{B}$

(b) $cos\ \hat{D}$

(c) $tan\ \hat{B}$

3. In $\triangle MNP$, $\hat{N} = 90°$, $MP = 20$ and $\hat{P} = 40°$. Calculate NP and MN (correct to 2 decimal places).

Solutions to Exercise 7 - 3

1. (a) $sin\ 37° = \frac{a}{62}$
   $a = 62 \times sin\ 37°$
   $a = 37{,}31$ units
   (b) $tan\ 23° = \frac{b}{21}$
   $b = 21 \times tan\ 23°$
   $b = 8{,}91$ units
   (c) $cos\ 55° = \frac{c}{19}$
   $c = 19 \times cos\ 55°$
   $c = 10{,}90$ units
   (d) $cos\ 49° = \frac{d}{33}$
   $d = 33 \times cos\ 49°$
   $d = 21{,}65$ units
   (e) $sin\ 17° = \frac{12}{e}$
   $e = \frac{12}{sin\ 17°}$
   $e = 41{,}04$ units
   (f) $cos\ 22° = \frac{31}{f}$
   $f = \frac{31}{cos\ 22°}$
   $f = 33{,}43$ units
   (g) $cos\ 23° = \frac{g}{32}$
   $g = 32 \times cos\ 23°$
   $g = 29{,}46$ units
   (h) $sin\ 30° = \frac{h}{20}$
   $h = 20 \times sin\ 30°$
   $h = 10{,}00$ units

2. (a) $sin\ \hat{B} = \frac{AC}{AB} = \frac{AD}{BD}$
   (b) $cos\ \hat{D} = \frac{AD}{BD} = \frac{CD}{AD}$
   (c) $tan\ \hat{B} = \frac{AC}{BC} = \frac{AD}{AB}$

3. $sin\ \hat{P} = \frac{MN}{MP}$
   $sin\ 40° = \frac{MN}{20}$
   $\therefore MN = 20\ sin\ 40°$
   $= 12{,}86$ units
   $cos\ \hat{P} = \frac{NP}{MP}$
   $cos\ 40; \circ = \frac{NP}{20}$
   $\therefore NP = 20\ cos\ 40°$
   $= 15{,}32$ units

Exercise 7 - 4

1. Determine the angle (correct to 1 decimal place):

   (a) $tan\ \theta = 1{,}7$
   (b) $sin\ \theta = 0{,}8$
   (c) $cos\ \alpha = 0{,}32$
   (d) $tan\ \theta = 5\frac{3}{4}$
   (e) $sin\ \theta = \frac{2}{3}$
   (f) $cos\ \gamma = 1{,}2$
   (g) $4\ cos\ \theta = 3$
   (h) $cos\ 4\theta = 0{,}3$
   (i) $sin\ \beta + 2 = 2{,}65$
   (j) $sin\ \theta = 0{,}8$
   (k) $3\ tan\ \beta = 1$
   (l) $sin\ 3\alpha = 1{,}2$

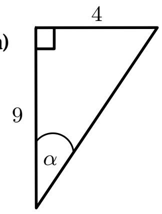
(m) $tan\ \frac{\theta}{3} = sin\ 48°$

(n) $\frac{1}{2}\ cos\ 2\beta = 0{,}3$

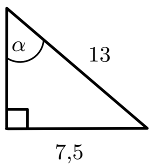
(o) $2\ sin\ 3\theta + 1 = 2{,}6$

2. Determine α in the following right-angled triangles:

(a)



(b)



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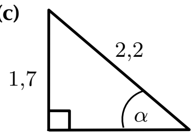

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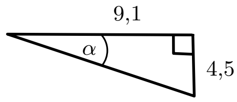

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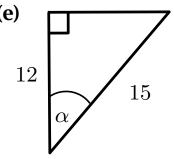
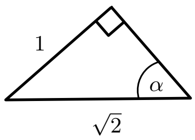


(f)



Solutions to Exercise 7 - 4

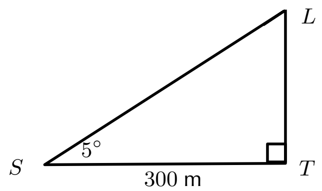
1. **(a)** $\theta = 59{,}5°$
   (b) $\theta = 53{,}1°$
   (c) $\theta = 71{,}3°$
   (d) $\theta = 80{,}1°$
   (e) $\theta = 41{,}8°$
   (f) No solution
   (g) $\theta = 41{,}4°$
   (h) $\theta = 18{,}1°$
   (i) $\theta = 40{,}5°$
   (j) $\theta = 53{,}1°$
   (k) $\theta = 18{,}4°$
   (l) No solution
   (m) $\theta = 109{,}8°$
   (n) $\theta = 26{,}6°$
   (o) $\theta = 17{,}7°$

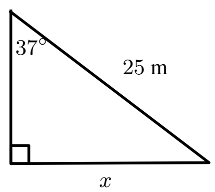
2. **(a)** $tan\ \alpha = \frac{4}{9} = 24{,}0°$
   (b) $sin\ \alpha = \frac{7{,}5}{13} = 35{,}2°$
   (c) $sin\ \alpha = \frac{1{,}7}{2{,}2} = 50{,}6°$
   (d) $tan\ \alpha = \frac{4{,}5}{9{,}1} = 26{,}3°$
   (e) $cos\ \alpha = \frac{12}{15} = 37{,}0°$
   (f) $sin\ \alpha = \frac{1}{\sqrt{2}} = 45{,}0°$

Exercise 7 - 5

1. A boy flying a kite is standing 30 m from a point directly under the kite. If the kite's string is 50 m long, find the angle of elevation of the kite.
2. What is the angle of elevation of the sun when a tree 7,15 m tall casts a shadow 10,1 m long?
3. From a distance of 300 m, Susan looks up at the top of a lighthouse. The angle of elevation is $5°$. Determine the height of the lighthouse to the nearest metre.
4. A ladder of length 25 m is resting against a wall, the ladder makes an angle $37°$ to the wall. Find the distance between the wall and the base of the ladder.

Solutions to Exercise 7 - 5

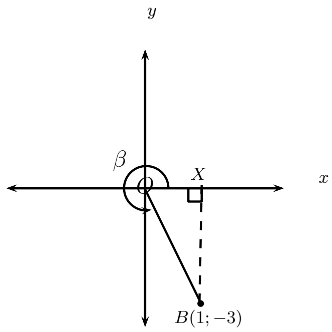
1. $cos\ x = \frac{30}{50}$
   $\therefore x = 53{,}13°$
2. $tan\ x = \frac{7{,}15}{10{,}1}$
   $\therefore x = 35{,}3°$
3. 

   In $\triangle LTS$
   $tan\ \hat{S} = \frac{LT}{ST}$
   $\therefore LT = 300\ tan\ 5°$
   $= 26$ m
4. 

   $sin\ 37° = \frac{x}{25}$
   $x = 25\ sin\ 37°$
   $x = 15$ m

Exercise 7 - 6

1. B is a point in the Cartesian plane. Determine without using a calculator:
   (a) OB
   (b) $cos\ \beta$
   (c) $cosec\ \beta$
   (d) $tan\ \beta$

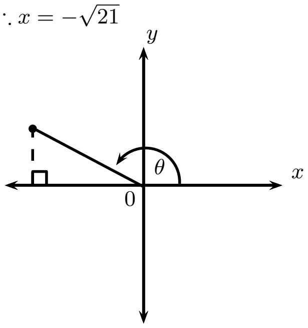


2. If $sin\ \theta = 0{,}4$ and θ is an obtuse angle, determine:
   (a) $cos\ \theta$
   (b) $\sqrt{21} tan\ \theta$
3. Given $tan\ \theta = \frac{t}{2}$, where $0° \leq \theta \leq 90°$. Determine the following in terms of t:
   (a) $sec\ \theta$
   (b) $cot\ \theta$
   (c) $cos^2\ \theta$
   (d) $tan^2\ \theta - sec^2\ \theta$

Solutions to Exercise 7 - 6

1. (a) In $\triangle BOX$
   $OB^2 = OX^2 + XB^2$
   $OB^2 = 1^2 + 3^2$
   $OB = \sqrt{10}$ units
   (b) $cos\ \beta = \frac{x}{r} = \frac{1}{\sqrt{10}}$
   (c) $cosec\ \beta = \frac{r}{y} = \frac{\sqrt{10}}{-3}$
   (d) $tan\ \beta = \frac{y}{x} = \frac{-3}{1} = -3$
2. $sin\ \theta = 0{,}4 = \frac{4}{10} = \frac{2}{5}$
   $\therefore y = 2;\ r = 5$

$\therefore x^2 = r^2 - y^2$
$= 5^2 - 2^2$
$= 21$
$\therefore x = \pm\sqrt{21}$
but angle is in III quadrant , therefore x is negative
$\therefore x = -\sqrt{21}$



(a) $cos\ \theta = \frac{x}{r} = \frac{-\sqrt{21}}{5}$
(b) $\sqrt{21}\ tan\ \theta = \sqrt{21}\left(\frac{y}{x}\right) = \sqrt{21}\left(\frac{2}{-\sqrt{21}}\right) = -2$

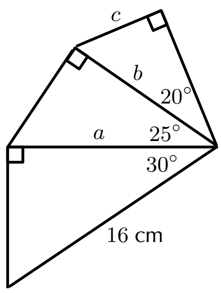
3. **(a)** $sec\ \theta = \frac{\sqrt{t^2+4}}{2}$
   (b) $cot\ \theta = \frac{2}{t}$
   (c) $cos^2\ \theta = (\frac{2}{\sqrt{t^2+4}})^2 = \frac{4}{t^2+4}$
   (d) $tan^2\ \theta - sec^2\ \theta$
   $= \frac{t^2}{4} - (\frac{\sqrt{t^2+4}}{2})^2$
   $= \frac{t^2}{4} - (\frac{t^2+4}{4})$
   $= \frac{t^2-t^2-4}{4}$
   $= -1$

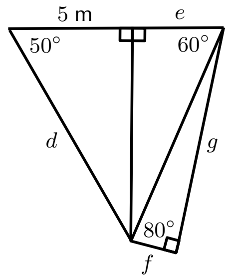
End of Chapter Exercises

1. Without using a calculator determine the value of

$$sin\ 60^\circ cos\ 30^\circ - cos\ 60^\circ sin\ 30^\circ + tan\ 45^\circ$$

2. If $3\ tan\ \alpha = -5$ and $0^\circ < \alpha < 270^\circ$, use a sketch to determine:
   (a) $cos\ \alpha$
   (b) $tan\ ^2\ \alpha - sec\ ^2\ \alpha$
3. Solve for θ if θ is a positive, acute angle:
   (a) $2\ sin\ \theta = 1{,}34$
   (b) $1 - tan\ \theta = -1$
   (c) $cos\ 2\theta = sin\ 40^\circ$
   (d) $\dfrac{sin\ \theta}{cos\ \theta} = 1$
4. Calculate the unknown lengths in the diagrams below:

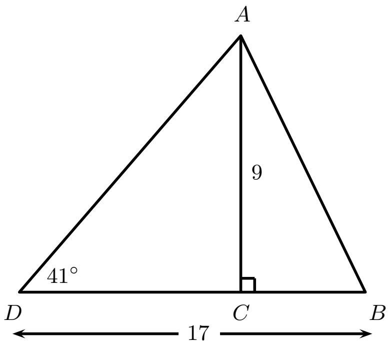




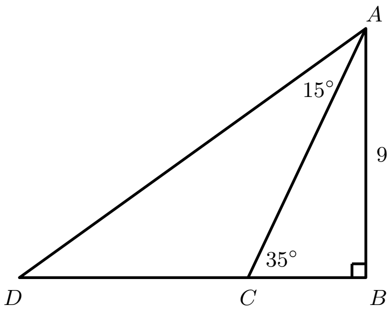
5. In $\triangle PQR$, $PR = 20$ cm, $QR = 22$ cm and $P\hat{R}Q = 30°$. The perpendicular line from P to QR intersects QR at X. Calculate:
   (a) the length XR
   (b) the length PX
   (c) the angle $Q\hat{P}X$

6. A ladder of length 15 m is resting against a wall, the base of the ladder is 5 m from the wall. Find the angle between the wall and the ladder.

7. In the following triangle find the angle $A\hat{B}C$:



8. In the following triangle find the length of side CD:



9. Given $A(5;0)$ and $B(11;4)$, find the angle between the line through A and B and the x-axis.

10. Given $C(0;-13)$ and $D(-12;14)$, find the angle between the line through C and D and the y-axis.

11. A right-angled triangle has hypotenuse 13 mm. Find the length of the other two sides if one of the angles of the triangle is $50°$.

12. One of the angles of a rhombus with perimeter 20 cm is $30°$.

(a) Find the sides of the rhombus.

(b) Find the length of both diagonals.

13. Captain Jack is sailing towards a cliff with a height of 10 m.

(a) The distance from the boat to the top of the cliff is 30 m. Calculate the angle of elevation from the boat to the top of the cliff (correct to the nearest integer).

(b) If the boat sails 7 m closer to the cliff, what is the new angle of elevation from the boat to the top of the cliff?

14. Given the points, $E(5;0)$, $F(6;2)$ and $G(8;-2)$, find the angle $F\hat{E}G$.

15. A triangle with angles $40°$, $40°$ and $100°$ has a perimeter of 20 cm. Find the length of each side of the triangle.

Solutions to End of Chapter Exercises

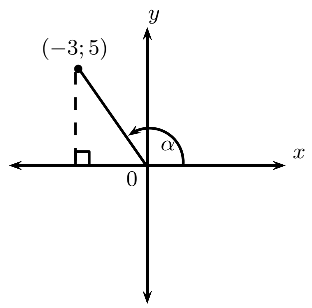
1. $sin\ 60° cos\ 30° - cos\ 60° sin\ 30° + tan\ 45°$

$= \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2} \times \frac{1}{2}\right) + 1$

$= \frac{3}{4} - \frac{1}{4} + 1$

$= \frac{2}{4} + 1$

$= 1\frac{1}{2}$



2.

$3\ tan\ \alpha = -5$

$tan\ \alpha = \frac{-5}{3}$

$\therefore x = -3;\ y = 5$

$\therefore r^2 = x^2 + y^2$

$= (-3)^2 + (5)^2$

$= 34$

$\therefore r = \sqrt{34}$

(a) $cos\ \alpha = \frac{x}{r} = \frac{-3}{\sqrt{34}}$

(b) $tan^2\alpha - sec^2\alpha$

$= \left(\frac{y}{x}\right)^2 - \left(\frac{r}{x}\right)^2$

$= \left(\frac{5}{-3}\right)^2 - \left(\frac{\sqrt{34}}{-3}\right)^2$

$= \frac{25}{9} - \frac{34}{9}$

$= \frac{-9}{9}$
$= -1$

3. (a) $2\ sin\ \theta = 1{,}34$
$sin\ \theta = \frac{1{,}34}{2}$
$\therefore \theta = 42{,}07°$

(b) $1 - tan\ \theta = -1$
$-\ tan\ \theta = -2$
$tan\ \theta = 2$
$\therefore \theta = 63{,}43°$

(c) $cos\ 2\theta = sin\ 40°$
$cos\ 2\theta = 0{,}64$
$2\theta = 50°$
$\theta = 25°$

(d) $\dfrac{sin\ \theta}{cos\ \theta} = 1$
$\therefore tan\ \theta = 1$
$\therefore \theta = 45°$

4. (a) $cos\ 30° = \frac{a}{16}$
$a = 16\ cos\ 30°$
$a = 13{,}86$ cm

(b) $cos\ 25° = \frac{b}{13{,}86}$
$\therefore b = 12{,}56$ cm

(c) $sin\ 20° = \frac{c}{12{,}56}$
$\therefore c = 4{,}30$ cm

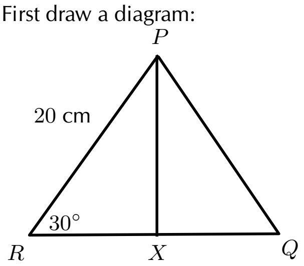
(d) $cos\ 50° = \frac{5}{d}$
$\therefore d = 7{,}78$ cm

(e) By Pythagoras the third side is
$\sqrt{5^2 + 7{,}779^2} = 9{,}246$ cm
$\therefore tan\ 60° = \frac{9{,}247}{e}$
$\therefore e = 5{,}34$ cm

(f) By Pythagoras the third side is
$\sqrt{5{,}339^2 + 7{,}779^2} = 9{,}435$ cm
$\therefore f = \sqrt{9{,}345^2 - 1{,}648^2} = 9{,}2$ cm

(g) $tan\ 80° = \frac{9{,}435}{g}$
$\therefore g = 1{,}65$ cm

5. First draw a diagram:

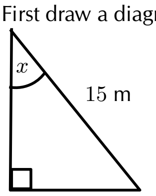


(a) $cos\ 30° = \frac{XR}{20}$
$XR = 20\ cos\ 30°$
$XR = 17{,}32$ cm

(b) $sin\ 30° = \frac{PX}{20}$
$PX = 20\ sin\ 30°$
$PX = 10$ cm

(c) $tan\ Q\hat{P}X = \frac{4{,}68}{10}$
$Q\hat{P}X = 25{,}08°$

6. First draw a diagram:



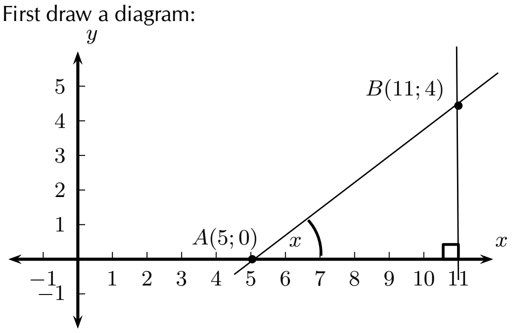
$sin\ x = \frac{5}{15}$
$\therefore x = 19{,}47°$

7. $tan\ 41° = \frac{9}{DC}$
$DC = \frac{9}{tan\ 41°}$
$= 10{,}35$ units

$BC = BD - DC = 6{,}65$ units
$tan\ A\hat{B}C = \dfrac{9}{6{,}65}$
$\therefore A\hat{B}C = 53{,}54°$

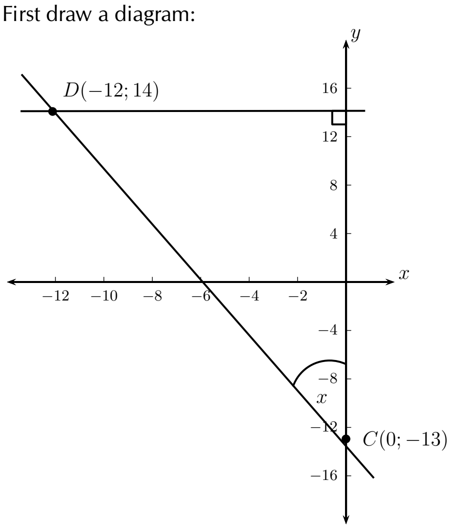
8. $C\hat{A}B = 180° - 90° - 35° = 55°$
$\therefore D\hat{A}B = 15° + 55° = 70°$
$tan\ 35° = \frac{9}{BC}$
$\therefore BC = 12{,}85$ units
$tan\ 70° = \frac{BD}{9}$
$BD = 24{,}73$ units
$CD = BD - BC = 11{,}88$ units

9. First draw a diagram:



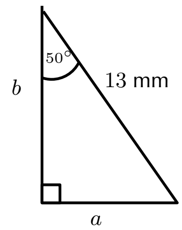
Note that the distance from B to the x-axis is 4 units and that the distance AC ia $11 - 5 = 6$ units.
$tan\ x = \frac{4}{6}$
$\therefore x = 33{,}69°$

10. First draw a diagram:



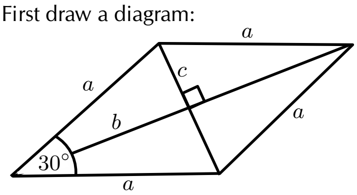
The distance from D to the y-axis is 12 units. The distance from C to the point where the perpendicular line from D intercepts the y-axis is $14 - (-13) = 27$ units.
$tan\ x = \frac{12}{27}$
$\therefore x = 23{,}96°$

11. First draw a diagram:



$sin\ 50° = \frac{a}{13}$
$a = 13\ sin\ 50°$
$\therefore a = 9{,}96$ mm
Use the theorem of Pythagoras to find the other side:
$b^2 = c^2 - a^2$
$= 13^2 - 9{,}96^2$
$\therefore b = \sqrt{69{,}7984}$
$= 8{,}35$ mm

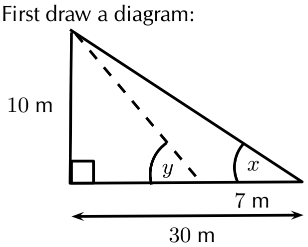
12. First draw a diagram:



(a) The perimeter is found by adding each side together. All the sides are equal in length therefore perimeter$= 4a$
$20 = 4a$
$\therefore a = 5$ cm

(b) The diagonals of a rhombus bisect the angle, so working in one of the small triangles we can use trigonometric identities to find b:
$cos\ 15° = \frac{b}{5}$
$\therefore b = 4{,}83$ cm
By Pythagoras $c^2 = a^2 - b^2$
$\therefore c = \sqrt{25 - 23{,}33}$
$c = 1{,}29$ cm
Since the diagonals bisect each other we know that the total length of each diagonal is either $2b$ or $2c$, depending on which diagonal we examine.
The one diagonal $= 4{,}8 \times 2 = 9{,}6$ cm
The other diagonal $= 1{,}29 \times 2 = 2{,}58$ cm

13. First draw a diagram:

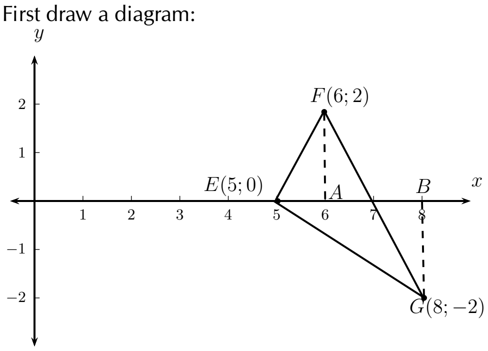


(a) $tan\ x = \frac{10}{30}$
$\therefore x = 18°$

(b) $tan\ y = \frac{10}{23}$

$\therefore y = 23°$

14. First draw a diagram:



To find $F\hat{E}G$ we inspect triangles FEA and GEB in turn.

In $\triangle FEA$

$tan\ F\hat{E}A = \frac{2}{1}$

$\therefore F\hat{E}A = 63{,}43°$

In $\triangle GEB$

$tan\ G\hat{E}B = \frac{2}{3}$

$\therefore G\hat{E}B = 33{,}69°$

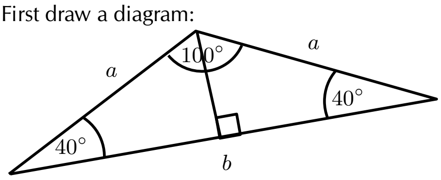
Add these two angles together to get the value of $F\hat{E}G$:

$F\hat{E}G = F\hat{E}B + G\hat{E}B$

$= 33{,}69° + 63{,}43°$

$\therefore F\hat{E}G = 97{,}12°$

15. First draw a diagram:



Construct a perpendicular bisector to create a right-angled triangle.

Therefore we have:

$2a + b = 20$

$b = 2(10 - a)$

$\therefore cos\ 50° = \frac{\frac{b}{2}}{a}$

$0{,}77 = \frac{\frac{2(10-a)}{2}}{a}$

$0{,}77 = \frac{10 - a}{a}$

$0{,}77a = 10 - a$

$\therefore a = 5{,}56$ cm

$\therefore b = 2(10 - 5{,}56) = 8{,}7$ cm

Analytical geometry 8

Exercise 8 - 1

1. Find the length of AB if:
   (a) $A(2;7)$ and $B(-3;5)$
   (b) $A(-3;5)$ and $B(-9;1)$
   (c) $A(x;y)$ and $B(x+4;y-1)$
2. The length of $CD = 5$. Find the missing coordinate if:
   (a) $C(6;-2)$ and $D(x;2)$
   (b) $C(4;y)$ and $D(1;-1)$

Solutions to Exercise 8 - 1

1. **(a)**
$$\begin{aligned} d_{AB} &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ &= \sqrt{(2-(-3))^2+(7-5)^2} \\ &= \sqrt{(5)^2+(2)^2} \\ &= \sqrt{29} \end{aligned}$$

   (b)
$$\begin{aligned} d_{AB} &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ &= \sqrt{(-3-(-9))^2+(5-1)^2} \\ &= \sqrt{(6)^2+(4)^2} \\ &= \sqrt{52} \end{aligned}$$

   (c)
$$\begin{aligned} d_{AB} &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ &= \sqrt{(x-(x+4))^2+(y-(y-1))^2} \\ &= \sqrt{(x-x-4)^2+(y-y+1)^2} \\ &= \sqrt{(-4)^2+(1)^2} \\ &= \sqrt{17} \end{aligned}$$

2. **(a)**
$$\begin{aligned} d_{CD} &= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \\ 5 &= \sqrt{(6-x)^2+(-2-2))^2} \\ 5^2 &= 36-12x+x^2+16 \\ 0 &= x^2-12x+36-25+16 \\ 0 &= x^2-12x+27 \end{aligned}$$

$= (x-3)(x-9)$
Therefore $x = 3$ or $x = 9$
Check solution for $x = 3$:
$d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(6-3)^2 + (-2-2))^2}$
$= \sqrt{(3)^2 + (-4)^2}$
$= \sqrt{25}$
$= 5$
Solution is valid.
Check solution for $x = 9$:
$d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(6-9)^2 + (-2-2))^2}$
$= \sqrt{(-3)^2 + (-4)^2}$
$= \sqrt{25}$
$= 5$
Solution is valid.

(b) $d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$5 = \sqrt{(4-1)^2 + (y+1))^2}$
$5^2 = 9 + y^2 + 2y + 1$
$0 = y^2 + 2y + 1 + 9 - 25$
$0 = y^2 + 2y - 15$
$= (y-3)(y+5)$
Therefore $y = 3$ or $y = -5$
Check solution for $y = 3$:
$d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(4-1)^2 + (3+1))^2}$
$= \sqrt{3^2 + 4^2}$
$= \sqrt{25}$
$= 5$
Solution is valid.
Check solution for $y = -5$:
$d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(4-1)^2 + (-5+1)^2}$
$= \sqrt{(3)^2 + (-4)^2}$
$= \sqrt{25}$
$= 5$
Solution is valid.

Exercise 8 - 2

1. Find the gradient of AB if

(a) $A(7;10)$ and $B(-4;1)$
(b) $A(-5;-9)$ and $B(3;2)$
(c) $A(x-3;y)$ and $B(x;y+4)$

2. If the gradient of $CD = \frac{2}{3}$, find p given
   (a) $C(16;2)$ and $D(8;p)$
   (b) $C(3;2p)$ and $D(9;14)$

Solutions to Exercise 8 - 2

1. (a) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{-1-10}{-4-7}$
   $= \frac{-11}{-11}$
   $= 1$

   (b) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{2-(-9)}{3-(-5)}$
   $= \frac{11}{8}$

   (c) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{y+4-y)}{x-(x-3)}$
   $= \frac{4}{3}$

2. (a) $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$
   $\frac{2}{3} = \frac{p-2}{8-16}$
   $\frac{2}{3} \times (-8) = p-2$
   $\frac{-16+6}{3} = p$
   $\frac{-10}{3} = p$

   (b) $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$
   $\frac{2}{3} = \frac{14-2p}{9-3}$
   $\frac{2}{3} \times (6) = 14-2p$
   $4-14 = -2p$
   $\frac{-10}{-2} = p$
   $5 = p$

Exercise 8 - 3

1. Determine whether AB and CD are parallel, perpendicular or neither if:
   (a) $A(3;-4)$, $B(5;2)$, $C(-1;-1)$, $D(7;23)$
   (b) $A(3;-4)$, $B(5;2)$, $C(-1;-1)$, $D(0;-4)$
   (c) $A(3;-4)$, $B(5;2)$, $C(-1;-1)$, $D(1;2)$

2. Determine whether the following points lie on the same straight line:
   (a) $E(0;3)$, $F(-2;5)$, $G(2;1)$
   (b) $H(-3;-5)$, $I(-0;0)$, $J(6;10)$
   (c) $K(-6;2)$, $L(-3;1)$, $M(1;-1)$

3. Points $P(-6;2)$, $Q(2;-2)$ and $R(-3;r)$ lie on a straight line. Find the value of r.

4. Line PQ with $P(-1;-7)$ and $Q(q;0)$ has a gradient of 1. Find q.

Solutions to Exercise 8 - 3

1. (a) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{2-(-4)}{5-3}$
   $= \frac{6}{2}$
   $= 3$
   And,
   $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{23-(-1)}{7-(3)}$
   $= \frac{24}{8}$
   $= 3$
   So $m_{AB} = m_{CD}$
   Therefore $AB \parallel CD$.
   (b) $m_{AB} = 3$
   And,
   $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{-4-(-1)}{0-(-1)}$
   $= \frac{-3}{1}$
   $= -3$
   So $m_{AB} \neq m_{CD}$
   Therefore AB is not parallel to CD.
   And $m_{AB} \times \frac{1}{m_{CD}} \neq -1$
   Therefore AB and CD are not perpendicular.
   (c) $m_{AB} = 3$
   And,
   $m_{CD} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{2-(-1)}{1-(-1)}$
   $= \frac{3}{2}$
   So $m_{AB} \neq m_{CD}$
   Therefore AB is not parallel to CD.
   And $m_{AB} \times \frac{1}{m_{CD}} \neq -1$
   Therefore AB and CD are not perpendicular.

2. (a) $m_{EF} = \frac{y_2-y_1}{x_2-x_1}$
   $= \frac{5-3}{-2-0}$
   $= \frac{2}{-2}$
   $= -1$

And,
$m_{FG} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1-5}{2-(-2)}$
$= \frac{-4}{4}$
$= -1$
So $m_{HI} = m_{FG}$ and F is a common point,
Therefore E, F and G are collinear.

(b) $m_{EF} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0-(-5)}{0-(-3)}$
$= \frac{5}{3}$
And,
$m_{IJ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{10-0}{6-0}$
$= \frac{10}{6}$
$= \frac{5}{3}$
So $m_{HI} = m_{IJ}$ and I is a common point,
Therefore H, I and J are collinear.

(c) $m_{KL} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1-2)}{-3-(-6)}$
$= -\frac{1}{3}$
And,
$m_{LM} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-1-1}{1-(-3)}$
$= \frac{-2}{4}$
$= -\frac{1}{2}$
So $m_{HI} \neq m_{IJ}$, therefore H, I and J are not collinear.

3. $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-2-5}{2-(-6)}$
$= \frac{-4}{8}$
$= -\frac{1}{2}$
And,
$m_{QR} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{r-(-2)}{-3-2}$
$= \frac{r+2}{-5}$
$= -\frac{1}{2}$
So,
$-\frac{1}{2} = \frac{r+2}{-5}$
$(-1) \times (-5) = 2(r+2)$
$5 = 2r + 4$
$5 - 4 = 2r$
$1 = 2r$
$r = \frac{1}{2}$

4. $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$1 = \frac{0-(-7)}{q-(-1)}$
$q + 1 = 7$
$q = 6$

Exercise 8 - 4

1. Find the mid-points of the following lines:
   (a) $A(2;5)$, $B(-4;7)$
   (b) $C(5;9)$, $D(23;55)$
   (c) $E(x+2;y-1)$, $F(x-5;y-4)$
2. The mid-point M of PQ is $(3;9)$. Find P if Q is $(-2;5)$.
3. $PQRS$ is a parallelogram with the points $P(5;3)$, $Q(2;1)$ and $R(7;-3)$. Find point S.

Solutions to Exercise 8 - 4

1. (a) $M_{AB} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
   $= (\frac{2-4}{2}; \frac{5+7}{2})$
   $= (\frac{-2}{2}; \frac{12}{2})$
   $= (-1;6)$
   (b) $M_{CD} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
   $= (\frac{5+23}{2}; \frac{9+55}{2})$
   $= (\frac{28}{2}; \frac{64}{2})$
   $= (14;32)$
   (c) $M_{EF} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
   $= (\frac{x+2+x-5}{2}; \frac{y-1+y-4}{2})$
   $= (\frac{2x-3}{2}; \frac{2y-5}{2})$

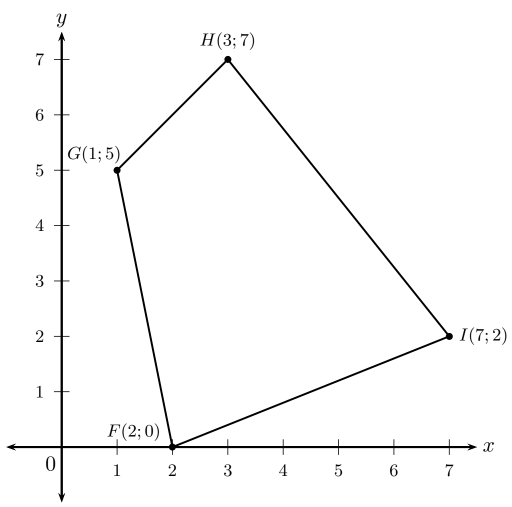
2. $M_{PQ} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
   $(3;9) = (\frac{x-2}{2}; \frac{y+5}{2})$
   Solve for x:
   $3 = \frac{x-2}{2}$
   $6 = x-2$
   $x = 8$
   Solve for y:
   $9 = \frac{y+5}{2}$
   $18 = y+5$
   $y = 13$
   Therefore $P(8;13)$

3. $M_{QR} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
   $= (\frac{2+7}{2}; \frac{1-3}{2})$

$= (\frac{9}{2}; \frac{-2}{2})$
$= (\frac{9}{2}; -1)$
Use mid-point M to find the coordinates of S:
$M_{QR} = (\frac{x_1+x_2}{2}; \frac{y_1+y_2}{2})$
$(\frac{9}{2}; -1) = (\frac{x+5}{2}; \frac{y+3}{2})$
Solve for x:
$\frac{9}{2} = \frac{x+5}{2}$
$9 = x + 5$
$x = 4$
Solve for y: $-1 = \frac{y+3}{2}$
$-2 = y + 3$
$y = -5$
Therefore $S(4; -5)$

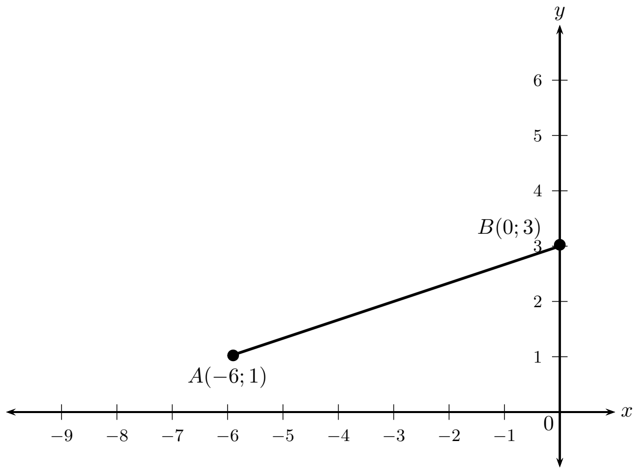
End of Chapter Exercises

1. Represent the following figures in the Cartesian plane:
   (a) Triangle DEF with $D(1; 2)$, $E(3; 2)$ and $F(2; 4)$
   (b) Quadrilateral $GHIJ$ with $G(2; -1)$, $H(0; 2)$, $I(-2; -2)$ and $J(1; -3)$
   (c) Quadrilateral $MNOP$ with $M(1; 1)$, $N(-1; 3)$, $O(-2; 3)$ and $P(-4; 1)$
   (d) Quadrilateral $WXYZ$ with $W(1; -2)$, $X(-1; -3)$, $Y(2; -4)$ and $Z(3; -2)$
2. In the diagram below, the vertices of the quadrilateral are $F(2; 0)$, $G(1; 5)$, $H(3; 7)$ and $I(7; 2)$.



   (a) Calculate the lengths of the sides of $FGHI$.
   (b) Are the opposite sides of $FGHI$ parallel?
   (c) Do the diagonals of $FGHI$ bisect each other?
   (d) Can you state what type of quadrilateral $FGHI$ is? Give reasons for your answer.

3. Consider a quadrilateral $ABCD$ with vertices $A(3;2)$, $B(4;5)$, $C(1;7)$ and $D(1;3)$.
   (a) Draw the quadrilateral.
   (b) Find the lengths of the sides of the quadrilateral.
4. $ABCD$ is a quadrilateral with vertices $A(0;3)$, $B(4;3)$, $C(5;-1)$ and $D(-1;-1)$.
   (a) Show by calculation that:
      i. $AD = BC$
      ii. $AB \parallel DC$
   (b) What type of quadrilateral is $ABCD$?
   (c) Show that the diagonals AC and BD do not bisect each other.
5. P, Q, R and S are the points $(-2;0)$, $(2;3)$, $(5;3)$, $(-3;-3)$ respectively.
   (a) Show that:
      i. $SR = 2PQ$
      ii. $SR \parallel PQ$
   (b) Calculate:
      i. PS
      ii. QR
   (c) What kind of quadrilateral is $PQRS$? Give reasons for your answer.
6. $EFGH$ is a parallelogram with vertices $E(-1;2)$, $F(-2;-1)$ and $G(2;0)$. Find the coordinates of H by using the fact that the diagonals of a parallelogram bisect each other.
7. $PQRS$ is a quadrilateral with points $P(0;-3)$, $Q(-2;5)$, $R(3;2)$ and $S(3;-2)$ in the Cartesian plane.
   (a) Find the length of QR.
   (b) Find the gradient of PS.
   (c) Find the mid-point of PR.
   (d) Is $PQRS$ a parallelogram? Give reasons for your answer.
8. $A(-2;3)$ and $B(2;6)$ are points in the Cartesian plane. $C(a;b)$ is the mid-point of AB. Find the values of a and b.
9. Consider triangle ABC with vertices $A(1;3)$, $B(4;1)$ and $C(6;4)$.
   (a) Sketch triangle ABC on the Cartesian plane.
   (b) Show that ABC is an isosceles triangle.
   (c) Determine the coordinates of M, the mid-point of AC.
   (d) Determine the gradient of AB.
   (e) Show that $D(7;-1)$ lies on the line that goes through A and B.
10. In the diagram, A is the point $(-6;1)$ and B is the point $(0;3)$



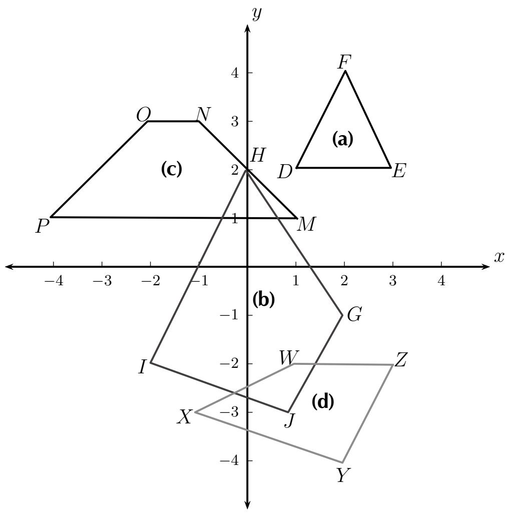
(a) Find the equation of line AB .

(b) Calculate the length of AB.

11. $\triangle PQR$ has vertices $P(1;8)$, $Q(8;7)$ and $R(7;0)$. Show through calculation that $\triangle PQR$ is a right angled isosceles triangle.

12. $\triangle ABC$ has vertices $A(-3;4)$, $B(3;-2)$ and $R(-5;-2)$. M is the midpoint of AC and N the midpoint of BC. Use $\triangle ABC$ to prove the midpoint theorem using analytical geometrical methods.

Solutions to End of Chapter Exercises



1.

2. (a) $d_{FG} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(1-2)^2 + (5-0)^2}$

$= \sqrt{(-1)^2 + (5)^2}$

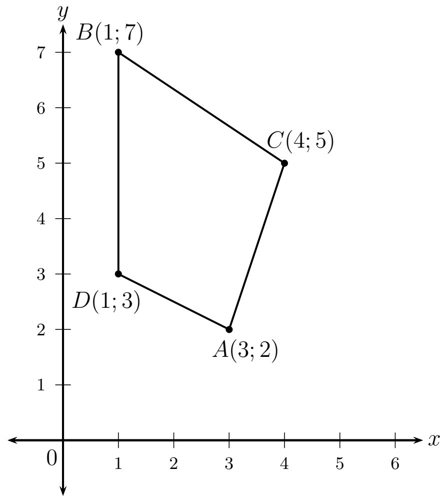
$= \sqrt{26}$

$d_{IH} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(7-3)^2 + (2-7)^2}$
$= \sqrt{(4)^2 + (-5)^2}$
$= \sqrt{41}$
Opposite sides FG and IH are not equal.
$d_{GH} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(3-1)^2 + (7-5)^2}$
$= \sqrt{(2)^2 + (2)^2}$
$= \sqrt{8}$
$d_{FI} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(2-7)^2 + (0-2)^2}$
$= \sqrt{(-5)^2 + (-2)^2}$
$= \sqrt{29}$
Opposite sides GH and FI are not equal.

(b) $m_{FG} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0-5}{2-1}$
$= \frac{-5}{1}$
$= -5$
$m_{IH} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2-7}{7-3}$
$= \frac{-5}{4}$
$= -\frac{-5}{4}$
Therefore $m_{FG} \neq m_{IH}$ and opposite sides are not parallel.
$m_{GH} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \dfrac{5-7}{1-3}$
$= \dfrac{-2}{-2}$
$= 1$
$m_{FI} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{0-2}{2-7}$
$= \frac{-2}{-5}$
$= \frac{2}{5}$
Therefore $m_{GH} \neq m_{FI}$ and opposite sides are not parallel.

(c) $M_{GI} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{1+7}{2}; \frac{5+2}{2})$
$= (\frac{8}{2}; \frac{7}{2})$
$= (4; \frac{7}{2})$
$M_{FH} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{3+2}{2}; \frac{7+0}{2})$
$= (\frac{5}{2}; \frac{7}{2})$
Therefore $M_{GI} \neq M_{FH}$ and diagonals do not bisect each other.

(d) This is an ordinary quadrilateral.

3. (a) 

(b) $d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(3-4)^2 + (2-5)^2}$
$= \sqrt{(-1)^2 + (-3)^2}$
$= \sqrt{10}$
$d_{BC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(4-1)^2 + (5-7)^2}$
$= \sqrt{(3)^2 + (-2)^2}$
$= \sqrt{13}$
$d_{CD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(1-1)^2 + (7-3)^2}$
$= \sqrt{0 + (4)^2}$
$= \sqrt{16}$
$= 4$
$d_{DA} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(1-3)^2 + (3-2)^2}$
$= \sqrt{(-2)^2 + (1)^2}$
$= \sqrt{5}$

4. (a) i. $d_{AD} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(0-(-1))^2 + (3-(-1))^2}$
$= \sqrt{(1)^2 + (4)^2}$
$= \sqrt{17}$
$d_{BC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(4-5)^2 + (3-(-1))^2}$
$= \sqrt{(-1)^2 + (4)^2}$
$= \sqrt{17}$
Therefore opposite sides AD and BC are equal.

ii. $m_{AB} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{3-3}{0-4}$
$= \frac{0}{-4}$
$= 0$

$m_{DC} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-1+1}{-1-5}$
$= \frac{0}{-6}$
$= 0$
Gradients are equal therefore opposite sides AB and DC are parallel.

(b) An isosceles trapezium; one pair of opposite sides equal in length and one pair of opposite sides parallel.

(c) $M_{AC} = (\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2})$
$= (\frac{0+5}{2}; \frac{3-1}{2})$
$= (\frac{5}{2}; \frac{2}{2})$
$= (\frac{5}{2}; 1)$
$M_{BD} = (\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2})$
$= (\frac{4-1}{2}; \frac{3-1}{2})$
$= (\frac{3}{2}; \frac{2}{2})$
$= (\frac{3}{2}; 1)$
Therefore diagonals do not bisect each other.

5. **(a)** **i.** $d_{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(-2-2)^2 + (0-3)^2}$
$= \sqrt{(-4)^2 + (-3)^2}$
$= \sqrt{25}$
$= 5$
$d_{SR} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(-3-5)^2 + (-3-3)^2}$
$= \sqrt{(-8)^2 + (-6)^2}$
$= \sqrt{100}$
$= 10$
Therefore $SR = 2PQ$.

ii. $m_{PQ} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{3-0}{2-(-2)}$
$= \frac{3}{4}$
$m_{SR} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-3-3}{-3-5}$
$= \frac{-6}{-8}$
$= \frac{3}{4}$
Therefore $m_{PQ} = m_{SR}$.

(b) **i.** $d_{PS} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(-2-(-3))^2 + (0-(-3))^2}$
$= \sqrt{(1)^2 + (3)^2}$
$= \sqrt{10}$

ii. $d_{QR} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(2-5)^2 + (3-3)^2}$
$= \sqrt{(-3)^2 + 0}$
$= \sqrt{9}$
$= 3$

(c) Trapezium.

6. $M_{EG} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{-1+2}{2}; \frac{2+0}{2})$
$= (\frac{1}{2}; 1)$
$M_{FH} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{-2+x}{2}; \frac{-1+y}{2})$
$(\frac{1}{2}; 1) = (\frac{-2+x}{2}; \frac{-1+y}{2})$
Solve for x:
$\frac{1}{2} = \frac{-2+x}{2}$
$1 = -2 + x$
$x = 3$
Solve for y:
$1 = \frac{-1+y}{2}$
$2 = -1 + y$
$y = 3$
Therefore $H(3; 3)$

7. **(a)** $d_{QR} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(-2 - 3)^2 + (5 - 2)^2}$
$= \sqrt{(-5)^2 + (3)^2}$
$= \sqrt{34}$

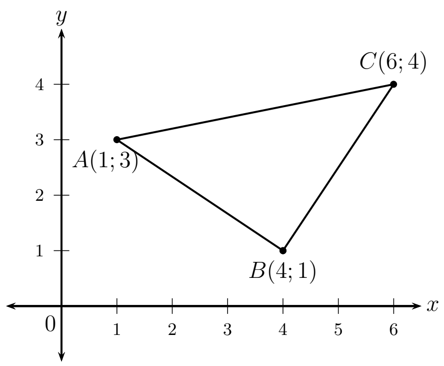
(b) $m_{PS} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-3+2}{0-3}$
$= \frac{-1}{-3}$
$= \frac{1}{3}$

(c) $M_{QR} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{0+3}{2}; \frac{-3+2}{2})$
$= (\frac{3}{2}; \frac{-1}{2})$

(d) $m_{RS} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-2-2}{3-3}$
$= \frac{-4}{0}$
$=$ undefined
And,
$m_{QR} = \dfrac{y_2 - y_1}{x_2 - x_1}$
$= \frac{2-5}{3-(-2)}$
$= \frac{-3}{5}$
Therefore $PQRS$ is not a parallelogram.

8. $M_{AB} = (\dfrac{x_1 + x_2}{2}; \dfrac{y_1 + y_2}{2})$
$= (\frac{-2-2}{2}; \frac{3+6}{2})$
$(a; b) = (0; \frac{9}{2})$
Therefore $a = 0$ and $b = \frac{9}{2}$.

9. (a)



(b) $d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(1-4)^2 + (3-1)^2}$
$= \sqrt{(-3)^2 + (2)^2}$
$= \sqrt{13}$
$d_{BC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(4-6)^2 + (1-4)^2}$
$= \sqrt{(-2)^2 + (-3)^2}$
$= \sqrt{13}$
$d_{AC} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(1-6)^2 + (3-4)^2}$
$= \sqrt{(-5)^2 + (-1)^2}$
$= \sqrt{26}$
Two sides of the triangle are equal in length, therefore $\triangle ABC$ is isosceles.

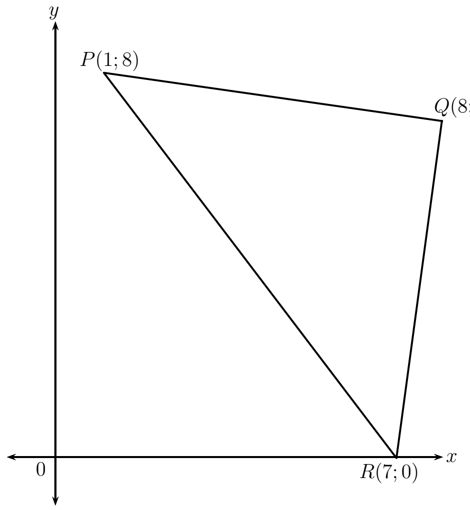
(c) $M_{AC} = (\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2})$
$= (\frac{1+6}{2}; \frac{3+4}{2})$
$= (\frac{7}{2}; \frac{7}{2})$

(d) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1-3}{4-1}$
$= \frac{-2}{3}$
$m_{BD} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-1-1}{7-4}$
$= \frac{-2}{3}$
$m_{AD} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{-1-3}{7-1}$
$= \frac{-2}{3}$
Therefore A,B and D are collinear.

10. (a) $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{1-3}{-6-0}$
$= \frac{-2}{-6}$
$= \frac{1}{3}$
Therefore equation of the line AB is $y = \frac{1}{3}x + 3$.

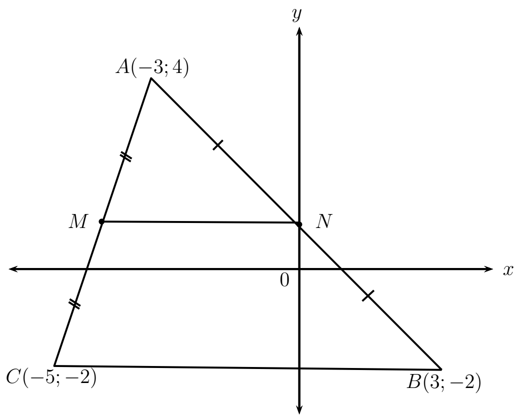
(b) $d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
$= \sqrt{(-6+0)^2 + (1-3)^2}$
$= \sqrt{(-6)^2 + (-2)^2}$

$= \sqrt{40}$



11.

$m_{PQ} = \frac{8-7}{1-8}$
$= -\frac{1}{7}$
$m_{RQ} = \frac{7-0}{8-7}$
$= \frac{7}{1}$
$m_{PR} \times m_{RQ} = -\frac{1}{7} \times \frac{7}{1}$
$= -1$
therefore $P\hat{Q}R = 90°$
$PQ = \sqrt{(1-8)^2 + (8-7)^2}$
$= \sqrt{49+1}$
$= \sqrt{50}$ units
$RQ = \sqrt{(8-7)^2 + (7-0)^2}$
$= \sqrt{1+49}$
$= \sqrt{50}$ units
Therefore $PQ = RQ$
Therefore $\triangle PQR$ is a right-angled, isosceles triangle.



12.

$M = (\frac{-3-5}{2}; \frac{4-2}{2})$
$= (-4; 1)$
$N = (\frac{-3+3}{2}; \frac{4-2}{2})$

$= (0;1)$
$m_{CB} = 0$ (parallel to x-axis)
$m_{MN} = 0$
MN parallel CB
Length $MN = 4$ units
Length $CB = 8$ units
therefore $MN = \frac{1}{2}CB$

Statistics 9

Exercise 9 - 1

1. Calculate the mean, median and mode of the following data sets:
   - **(a)** $2;\ 5;\ 8;\ 8;\ 11;\ 13;\ 22;\ 23;\ 27$
   - **(b)** $15;\ 17;\ 24;\ 24;\ 26;\ 28;\ 31;\ 43$
   - **(c)** $4;\ 11;\ 3;\ 15;\ 11;\ 13;\ 25;\ 17;\ 2;\ 11$
   - **(d)** $24;\ 35;\ 28;\ 41;\ 32;\ 49;\ 31$
2. The ages of 15 runners of the Comrades Marathon were recorded:

   $$31;\ 42;\ 28;\ 38;\ 45;\ 51;\ 33;\ 29;\ 42;\ 26;\ 34;\ 56;\ 33;\ 46;\ 41$$

   Calculate the mean, median and modal age.
3. In the first of a series of jars, there is 1 sweet. In the second jar, there are 3 sweets. The mean number of sweets in the first two jars is 2.
   - **(a)** If the mean number of sweets in the first three jars is 3, how many sweets are there in the third jar?
   - **(b)** If the mean number of sweets in the first four jars is 4, how many sweets are there in the fourth jar?
4. Find a set of five ages for which the mean age is 5, the modal age is 2 and the median age is 3 years.
5. Four friends each have some marbles. They work out that the mean number of marbles they have is 10. One friend leaves with 4 marbles. How many marbles do the remaining friends have together?

Solutions to Exercise 9 - 1

1. **(a)** Mean $= 13{,}2$; Median $= 11$; Mode $= 8$
   - **(b)** Mean $= 26$; Median $= 25$; Mode $= 24$
   - **(c)** Mean $= 11{,}2$; Median $= 11$; Mode $= 11$
   - **(d)** Mean $= 34{,}29$; Median $= 32$; No mode
2. Mean $= 38{,}3$; Median $= 38$; Mode $= 33$ and 42

3. (a) Let n_3 be the number of sweets in the third jar:
$\frac{1+3+n_3}{3} = 3$
$1 + 3 + n_3 = 9$
$n_3 = 5$

(b) Let n_4 be the number of sweets in the fourth jar:
$\frac{1+3+5+n_4}{4} = 4$
$9 + n_4 = 16$
$n_4 = 7$

4. Let the five different ages be x_1, x_2, x_3, x_4 and x_5.
Therefore the mean is
$\frac{x_1+x_2+x_3+x_4+x_5}{5} = 5$
$x_1 + x_2 + x_3 + x_4 + x_5 = 25$
The median value is at position 3, therefore $x_3 = 3$.
The mode is the age that occurs most often. There are at least 2 ages to be 2. The other ages would all have to be different. The four unknown ages can't all be 2, as that would not give us a mean of 5. Also since all calculations of mean, mode and median are done on ordered sets of data we can't have 3 ages being 2, because then the median age would not be 3. So we have
$2 + 2 + 3 + x_4 + x_5 = 25$
$18 = x_4 + x_5$
x_4 and x_5 can be any numbers that add up to 18 and are not the same, so 12 and 6 or 8 and 10 or 3 and 15, etc.
Possible data sets:
$2, 2, 3, 3, 15;\quad 2, 2, 3, 4, 14;\quad 2, 2, 3, 5, 13;\quad 2, 2, 3, 6, 12;\quad 2, 2, 3, 7, 11;\quad 2, 2, 3, 8, 10$
Note that the set of ages must be ordered, the median value must be 3 and there must be 2 ages of 2.

5. Let the number of marbles per friend be x_1, x_2, x_3 and x_4.
$\frac{x_1+x_2+x_3+x_4}{4} = 10$
$x_1 + x_2 + x_3 + x_4 = 40$
One friend leaves,
$x_1 + x_2 + x_3 = 40 - 4$
$x_1 + x_2 + x_3 = 36$
Therefore the remaining friends have 36 marbles.

Exercise 9 - 2

1. A class experiment was conducted and 50 learners were asked to guess the number of sweets in a jar. The following guesses were recorded:

56	49	40	11	33	33	37	29	30	59
21	16	38	44	38	52	22	24	30	34
42	15	48	33	51	44	33	17	19	44
47	23	27	47	13	25	53	57	28	23
36	35	40	23	45	39	32	58	22	40

(a) Draw up a grouped frequency table using the intervals
$10 < x \leq 20$; $20 < x \leq 30$; $30 < x \leq 40$; $40 < x \leq 50$; and $50 < x \leq 60$.

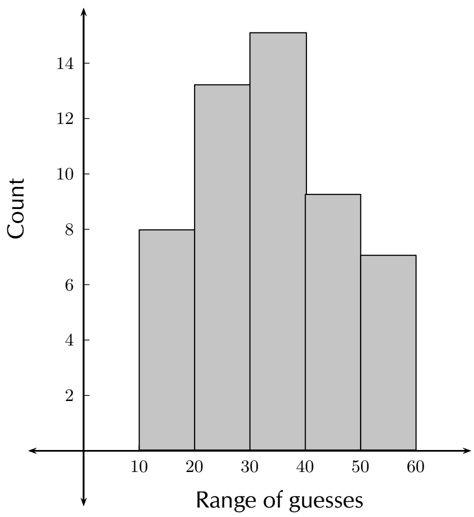
(b) Draw the histogram corresponding to the frequency table of the grouped data.

Solutions to Exercise 9 - 2

1. **(a)**

Group	**Freq**
$11 - 20$	6
$21 - 30$	13
$31 - 40$	15
$41 - 50$	9
$51 - 60$	7

(b)



Exercise 9 - 3

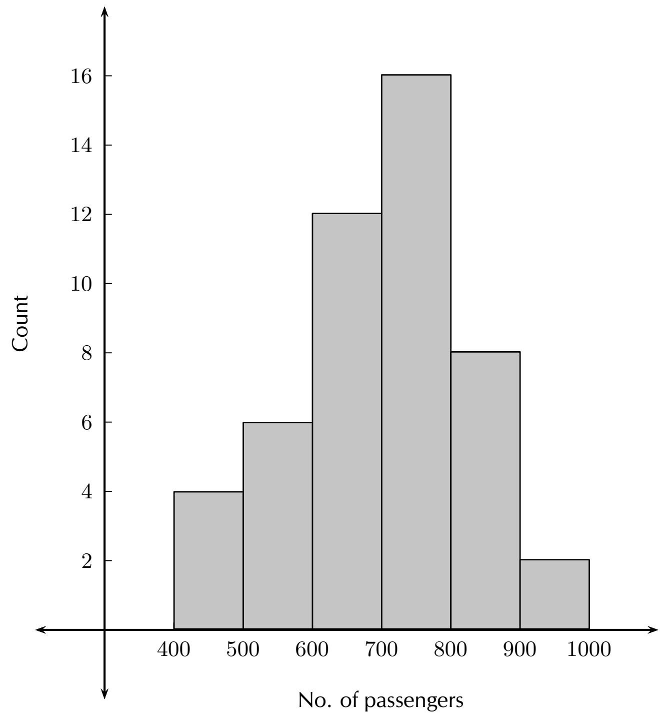
1. Consider the following grouped data and calculate the mean, the modal group and the median group.

Mass (kg)	Count
$40 < m \leq 45$	7
$45 < m \leq 50$	10
$50 < m \leq 55$	15
$55 < m \leq 60$	12
$60 < m \leq 65$	6

2. Find the mean, the modal group and the median group in this data set of how much time people needed to complete a game.

Time (s)	Count
$35 < t \leq 45$	5
$45 < t \leq 55$	11
$55 < t \leq 65$	15
$65 < t \leq 75$	26
$75 < t \leq 85$	19
$85 < t \leq 95$	13
$95 < t \leq 105$	6

3. The histogram below shows the number of passengers that travel in Alfred's minibus taxi per week.
   Calculate
   (a) the modal interval
   (b) the total number of passengers to travel in Alfred's taxi
   (c) an estimate of the mean
   (d) an estimate of the median
   (e) if it is estimated that every passenger travelled an average distance of 5 km, how much money would Alfred have made if he charged R $3{,}50$ per km?



Solutions to Exercise 9 - 3

1. Mean = 53; Modal group: $50 < m \leq 55$; Median group: $50 < m \leq 55$
2. Mean = 71,66; Modal group: $65 < t \leq 75$; Median group: $65 < t \leq 75$
3. **(a)** $700 < x \leq 800$
   (b) 33 600
   (c) 700
   (d) 750
   (e) R 588 000

Exercise 9 - 4

1. Find the range of the data set

$$\{1;\ 2;\ 3;\ 4;\ 4;\ 4;\ 5;\ 6;\ 7;\ 8;\ 8;\ 9;\ 10;\ 10\}$$

2. What are the quartiles of this data set?

$$\{3;\ 5;\ 1;\ 8;\ 9;\ 12;\ 25;\ 28;\ 24;\ 30;\ 41;\ 50\}$$

3. A class of 12 students writes a test and the results are as follows:

$$20;\ 39;\ 40;\ 43;\ 43;\ 46;\ 53;\ 58;\ 63;\ 70;\ 75;\ 91$$

   Find the range, quartiles and the interquartile range.

4. Three sets of data are given:
   - **Data set 1:** $\{9;\ 12;\ 12;\ 14;\ 16;\ 22;\ 24\}$
   - **Data set 2:** $\{7;\ 7;\ 8;\ 11;\ 13;\ 15;\ 16;\ 16\}$
   - **Data set 3:** $\{11;\ 15;\ 16;\ 17;\ 19;\ 19;\ 22;\ 24;\ 27\}$

   For each data set find:

   (a) the range
   (b) the lower quartile
   (c) the interquartile range
   (d) the semi-interquartile range
   (e) the median
   (f) the upper quartile

Solutions to Exercise 9 - 4

1. Range: $10 - 1 = 9$
2. $Q_1 = 6{,}2;\ Q_2 = 18;\ Q_3 = 29$
3. Range: $91 - 20 = 71$
   $Q_1 = 41{,}5$; $Q_2 = 49{,}5$; $Q_3 = 66{,}5$
   Interquartile range: $66{,}5 - 41{,}5 = 25$
4. **(a)** Range: $24 - 9 = 15;\quad 16 - 7 = 9;\quad 27 - 11 = 16$
   (b) Q_1: 12; 7,5; 15,5
   (c) Interquartile range: 10; 8; 7,5
   (d) Semi-interquartile range: 5; 4; 3,75
   (e) Median: 14; 12; 19
   (f) Q_3: 22; 15,5; 23

Exercise 9 - 5

1. Lisa is working in a computer store. She sells the following number of computers each month:

$$\{27;\ 39;\ 3;\ 15;\ 43;\ 27;\ 19;\ 54;\ 65;\ 23;\ 45;\ 16\}$$

Give the five number summary and box-and-whisker plot of Lisa's sales.

2. Zithulele works as a telesales person. He keeps a record of the number of sales he makes each month. The data below show how much he sells each month.

$$\{49;\ 12;\ 22;\ 35;\ 2;\ 45;\ 60;\ 48;\ 19;\ 1;\ 43;\ 12\}$$

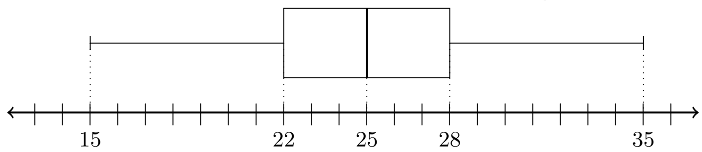
Give the five number summary and box-and-whisker plot of Zithulele's sales.

3. Hannah has worked as a florist for nine months. She sold the following number of wedding bouquets:

$$\{16;\ 14;\ 8;\ 12;\ 6;\ 5;\ 3;\ 5;\ 7\}$$

Give the five number summary of Hannah's sales.

4. Use the diagram below to determine the five number summary:

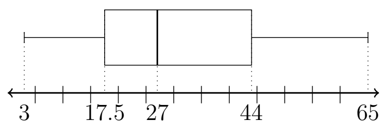


(a)

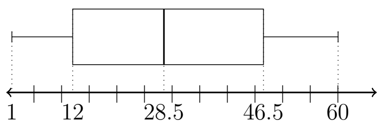
(b)

Solutions to Exercise 9 - 5

1. Minimum: 3
   Q_1: 17,5
   Median: 27
   Q_3: 44
   Maximum: 65

2. Minimum: 1
   Q_1: 12
   Median: 28,5
   Q_3: 46,5
   Maximum: 60



3. Minimum: 3
   Q_1: 5
   Median: 7
   Q_3: 13
   Maximum: 16

4. (a) Minimum: 15
   Q_1: 22
   Median: 25
   Q_3: 28
   Maximum: 35

   (b) Minimum: 88
   Q_1: 92
   Median: 98
   Q_3: 100
   Maximum: 101

End of Chapter Exercises

1. In a park, the tallest 7 trees have heights in metres of 41; 60; 47; 42; 44; 42; and 47. Find the median of their heights.
2. The students in Ndeme's class have the following ages: 5; 6; 7; 5; 4; 6; 6; 6; 7; 4. Find the mode of their ages.
3. An engineering company has designed two different types of engines for motorbikes. The two different motorbikes are tested for the time (in seconds) it takes for them to accelerate from 0 km/h to 60 km/h.

	Test 1	**Test 2**	**Test 3**	**Test 4**	**Test 5**	**Test 6**	**Test 7**	**Test 8**	**Test 9**	**Test 10**
Bike 1	1,55	1,00	0,92	0,80	1,49	0,71	1,06	0,68	0,87	1,09
Bike 2	0,9	1,0	1,1	1,0	1,0	0,9	0,9	1,0	0,9	1,1

   (a) Which measure of central tendency should be used for this information?
   (b) Calculate the measure of central tendency that you chose in the previous question, for each motorbike.
   (c) Which motorbike would you choose based on this information? Take note of the accuracy of the numbers from each set of tests.
4. In a traffic survey, a random sample of 50 motorists were asked the distance they drove

to work daily. This information is shown in the table below.

Distance (km)	Count
$0 < d \leq 5$	4
$5 < d \leq 10$	5
$10 < d \leq 15$	9
$15 < d \leq 20$	10
$20 < d \leq 25$	7
$25 < d \leq 30$	8
$30 < d \leq 35$	3
$35 < d \leq 40$	2
$40 < d \leq 45$	2

(a) Find the approximate mean of the data.

(b) What percentage of samples had a distance of

i. less than 16 km?

ii. more than 30 km?

iii. between 16 km and 30 km daily?

(c) Draw a histogram to represent the data

5. A company wanted to evaluate the training programme in its factory. They gave the same task to trained and untrained employees and timed each one in seconds.

Trained	121	137	131	135	130
	128	130	126	132	127
	129	120	118	125	134
Untrained	135	142	126	148	145
	156	152	153	149	145
	144	134	139	140	142

(a) Find the medians and quartiles for both sets of data.

(b) Find the interquartile range for both sets of data.

(c) Comment on the results.

(d) Draw a box-and-whisker diagram for each data set to illustrate the five number summary.

6. A small firm employs nine people. The annual salaries of the employers are:

R 600 000	R 250 000	R 200 000
R 120 000	R 100 000	R 100 000
R 100 000	R 90 000	R 80 000

(a) Find the mean of these salaries.

(b) Find the mode.

(c) Find the median.

(d) Of these three figures, which would you use for negotiating salary increases if you were a trade union official? Why?

Solutions to End of Chapter Exercises

1. Median $= \dfrac{41+60+47+42+44+42+47}{7}$
$= 44$

2. Mode $= 6$

3. **(a)** Mean and Mode. The mean will give us the average acceleration time, while the mode will give us the time that is most often obtained.

   (b) For bike 1 the mean is $1{,}02$ s and no mode, since there is no value that occurs more than once.
   For bike 2 the mean is $1{,}0$ s and there are two modes, $1{,}0$ and $0{,}9$.

   (c) It would be difficult to choose. Although bike 1 appears to do better than bike 2 from the mean, the data for bike 2 is less accurate than that for bike 1 (it only has 1 decimal place.) If we were to calculate the mean for bike 1 using only 1 decimal place we would get $0{,}9$ s. This would make bike 2 better. Also bike 2 produces more consistent numbers. So bike 2 would likely be a good choice, but more information or more accurate information should be obtained.

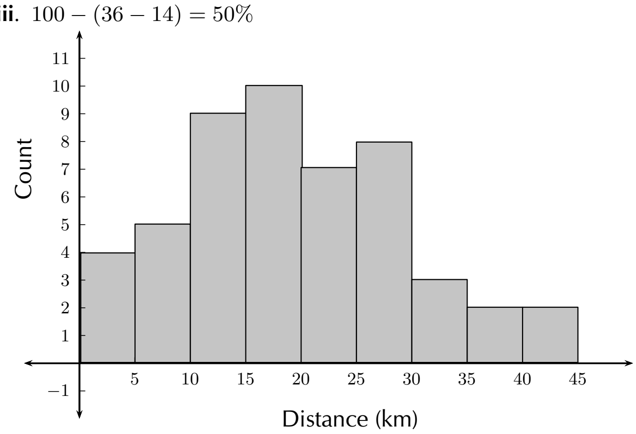
4. **(a)** Mean $= \dfrac{4(3)+5(8)+9(13)+10(18)+7(23)+8(28)+3(33)+2(38)+2(43)}{50}$
$= 19{,}9$

   (b) **i.** There were 18 drivers who drove less than 15 km.
   Therefore $\frac{18}{50} \times 100 = 38\%$

   ii. There were 7 drivers who drove less than 30 km.
   Therefore $\frac{7}{50} \times 100 = 14\%$

   iii. $100 - (36 - 14) = 50\%$

   (c)



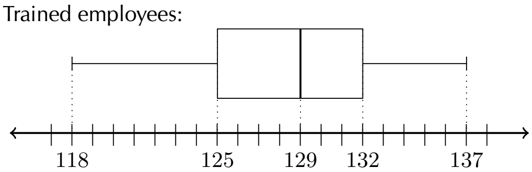
5. **(a)** First order the data sets for both trained and untrained employees.
   Trained: $118, 120, 121, 125, 126, 127, 128, 129, 130, 130, 131, 132, 134, 135, 137$
   Untrained: $126, 134, 135, 139, 140, 142, 142, 144, 145, 145, 148, 149, 152, 153, 156$

There are 15 values in each data set.
Position of the median is $\frac{15+1}{2} = 8$.
For the trained employees this is 129 and for the untrained employees this is 144.
Positions of the quartiles are $\frac{15}{4} = 3{,}75$.
For the trained employees: $Q_1 = 125$ and $Q_3 = 132$.
For the untrained employees: $Q_1 = 139$ and $Q_3 = 149$.

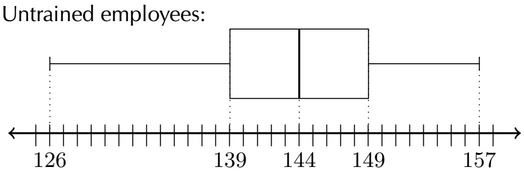
(b) Interquartile range for the trained employees: $Q_3 - Q_1 = 7$.
Interquartile range for the untrained employees: $Q_3 - Q_1 = 10$.

(c) The median of the untrained employees is higher than that of the trained employees. Also the untrained employees have a larger interquartile range than the trained employees. There is some evidence to suggest that the training programme may be working.

(d) Trained employees:



Untrained employees:



6. **(a)** Mean $= \frac{1\ 640\ 000}{9} = 182\ 222{,}22$

(b) Mode is R $100\ 000$.

(c) First order the data. To make the numbers easier to work with we will divide each one by $100\ 000$.
The ordered set is $80, 90, 100, 100, 100, 120, 200, 250, 600$.
The median is at position 5 and is R $100\ 000$.

(d) Either the mode or the median.
The mean is skewed (shifted) by the one salary of R $600\ 000$. The mode gives us a better estimate of what the employees are actually earning. The median also gives us a fairly accurate representation of what the employees are earning.

Probability 10

Exercise 10 - 1

1. A bag contains 6 red, 3 blue, 2 green and 1 white balls. A ball is picked at random. Determine the probability that it is:
   (a) red
   (b) blue or white
   (c) not green
   (d) not green or red
2. A playing card is selected randomly from a pack of 52 cards. Determine the probability that it is:
   (a) the 2 of hearts
   (b) a red card
   (c) a picture card
   (d) an ace
   (e) a number less than 4
3. Even numbers in the range 2 to 100 are written on cards. What is the probability of selecting a multiple of 5, if a card is drawn at random?

Solutions to Exercise 10 - 1

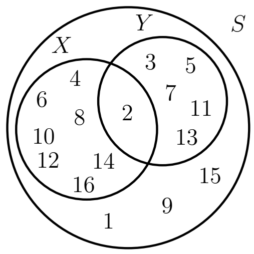
1. **(a)** $\frac{6}{12} = \frac{1}{2}$
   (b) $\frac{(3+1)}{12} = \frac{1}{3}$
   (c) $1 - (\frac{2}{12}) = \frac{5}{6}$
   (d) $1 - \frac{(2+6)}{12} = \frac{4}{12}$
   $= \frac{1}{3}$
2. **(a)** $\frac{1}{52}$ (only one in the deck)
   (b) $\frac{1}{2}$ (half the cards are red, half are black)
   (c) $\frac{3}{13}$ (for each suite of 13 cards, there are three picture cards: J, Q, K)
   (d) $\frac{4}{52} = \frac{1}{13}$ (four aces in the deck)
   (e) $\frac{3}{13}$ (for each suite of 13 cards, there are three cards less than 4: A, 2 and 3)
3. There are 50 cards. They are all even.
   All even numbers that are also multiples of 5 are multiples of 10 $(10, 20, \ldots, 100)$.

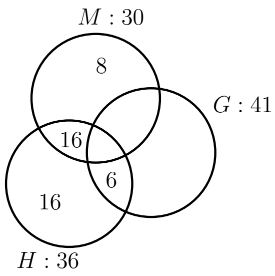
There are 10 of them.
Therefore, the probability is $\frac{10}{50} = \frac{1}{5}$.

Exercise 10 - 2

1. Let S denote the set of whole numbers from 1 to 16, X denote the set of even numbers from 1 to 16 and Y denote the set of prime numbers from 1 to 16. Draw a Venn diagram accurately depicting S, X and Y.
2. There are 79 Grade 10 learners at school. All of these take some combination of Maths, Geography and History. The number who take Geography is 41, those who take History is 36, and 30 take Maths. The number who take Maths and History is 16; the number who take Geography and History is 6, and there are 8 who take Maths only and 16 who take only History.
   (a) Draw a Venn diagram to illustrate all this information.
   (b) How many learners take Maths and Geography but not History?
   (c) How many learners take Geography only?
   (d) How many learners take all three subjects?
3. Pieces of paper labelled with the numbers 1 to 12 are placed in a box and the box is shaken. One piece of paper is taken out and then replaced.
   (a) What is the sample space, S?
   (b) Write down the set A, representing the event of taking a piece of paper labelled with a factor of 12.
   (c) Write down the set B, representing the event of taking a piece of paper labelled with a prime number.
   (d) Represent A, B and S by means of a Venn diagram.
   (e) Find
      i. $n(S)$
      ii. $n(A)$
      iii. $n(B)$

Solutions to Exercise 10 - 2

1. 

2. (a) 

(b) Each student must do exactly one of the following:
- Take only geography;
- Only take maths and/or history;

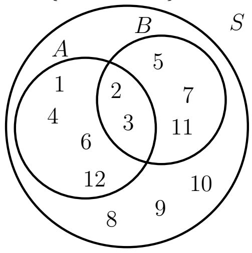
There are $30 + 36 - 16 = 50$ students doing the second one, therefore there must be $79 - 50 = 29$ students only doing geography.
Each student must do exactly one of:
- Only take geography;
- Only take maths;
- Take history;
- Take geography and maths, but not history;

There are $29, 8$ and 36 of the first three, so the answer to b) is:
$79 - 29 - 8 - 36 = 6$ students

(c) Calculated already: 29

(d) Each student must do exactly one of:
- Geography
- Only maths
- Only history
- Maths and history but not geography

Using the same method as before, the number of students in the last group is:
$79 - 41 - 8 - 16 = 14$
But, 16 students do maths and history, so there must be $16 - 14 = 2$ students who do all three.

3. (a) $S = \{1; 2; \ldots; 12\}$
(b) $A = \{1; 2; 3; 4; 6; 12\}$
(c) $B = \{2; 3; 5; 7; 11\}$
(d) 

(e) i. 12
ii. 6
iii. 5

Exercise 10 - 3

1. A box contains coloured blocks. The number of each colour is given in the following table.

Colour	Purple	Orange	White	Pink
Number of blocks	24	32	41	19

A block is selected randomly. What is the probability that the block will be:

(a) purple
(b) purple or white
(c) pink and orange
(d) not orange?

2. A small school has a class with children of various ages. The table gives the number of pupils of each age in the class.

	3 years old	4 years old	5 years old
Male	2	7	6
Female	6	5	4

If a pupil is selected at random what is the probability that the pupil will be:

(a) a female
(b) a 4 year old male
(c) aged 3 or 4
(d) aged 3 and 4
(e) not 5
(f) either 3 or female?

3. Fiona has 85 labelled discs, which are numbered from 1 to 85. If a disc is selected at random what is the probability that the disc number:

(a) ends with 5
(b) is a multiple of 3
(c) is a multiple of 6
(d) is number 65
(e) is not a multiple of 5
(f) is a multiple of 4 or 3
(g) is a multiple of 2 and 6
(h) is number 1?

Solutions to Exercise 10 - 3

1. Before we answer the questions we first work out how many blocks there are in total. This gives us the sample space
   $n(S) = 24 + 32 + 41 + 19$
   $= 116$
   (a) The probability that a block is purple is:
   $P(\text{purple}) = \frac{n(E)}{n(S)}$
   $P(\text{purple}) = \frac{24}{116}$
   $P(\text{purple}) = 0{,}21$
   (b) The probability that a block is either purple or white is:
   $P(\text{purple} \cup \text{white}) = P(\text{purple}) + P(\text{white}) - P(\text{purple} \cap \text{white})$
   $= \frac{24}{116} + \frac{41}{116} - \frac{24}{116} \times \frac{41}{116}$
   $= 0{,}64$
   (c) Since one block cannot be two colours the probability of this event is 0.
   (d) We first work out the probability that a block is orange:
   $P(\text{orange}) = \frac{32}{116} = 0{,}28$
   The probability that a block is not orange is:
   $P(\text{not orange}) = 1 - 0{,}28$
   $= 0{,}72$

2. We calculate the total number of pupils at the school:
   $6 + 2 + 5 + 7 + 4 + 6 = 30$
   (a) The total number of female children is $6 + 5 + 4 = 15$.
   The probability of a randomly selected child being female is:
   $P(\text{female}) = \frac{n(E)}{n(S)}$
   $P(\text{female}) = \frac{15}{30}$
   $P(\text{female}) = 0{,}5$
   (b) The probability of a randomly selected child being a 4 year old male is:
   $P(\text{male}) = \frac{7}{30}$
   $= 0{,}23$
   (c) There are $6 + 2 + 5 + 7 = 20$ children aged 3 or 4.
   The probability of a randomly selected child being either 3 or 4 is:
   $\frac{20}{30} = 0{,}67$
   (d) A child cannot be both 3 and 4, so the probability is 0.
   (e) This is the same as a randomly selected child being either 3 or 4 and so is $0{,}67$.
   (f) The probability of a child being either 3 or female is:
   $P(3 \cup \text{female}) = P(3) + P(\text{female}) - P(3 \cap \text{female})$
   $= \frac{10}{30} + 0{,}5 - \frac{10}{30} \times \frac{15}{30}$
   $= 0{,}67$

3. **(a)** The set of all discs ending with 5 is: $\{5; 15; 25; 35; 45; 55; 65; 75; 85\}$. This has 9 elements.
   The probability of drawing a disc that ends with 5 is:
   $P(5) = \frac{n(E)}{n(S)}$

$P(5) = \frac{9}{85}$
$P(5) = 0{,}11$

(b) The set of all discs that are multiples of 3 is: $\{3; 6; 9; 12; 15; 18; 21; 24; 27; 30; 33; 36; 39; 42; 45; 48; 51; 54; 57; 60; 63; 66; 69; 72; 75; 78; 81; 84\}$. This has 28 elements.
The probability of drawing a disc that is a multiple of 3 is: $P(3_m) = \frac{28}{85} = 0{,}33$

(c) The set of all discs that are multiples of 6 is: $\{6; 12; 18; 24; 30; 36; 42; 48; 54; 60; 66; 72; 78; 84\}$. This set has 14 elements. The probability of drawing a disc that is a multiple of 6 is:
$P(6_m) = \frac{14}{85}$
$= 0{,}16$

(d) There is only one element in this set and so the probability of drawing 65 is:
$P(65) = \frac{1}{85}$
$= 0{,}01$

(e) The set of all discs that is a multiple of 5 is: $\{5; 10; 15; 20; 25; 30; 35; 40; 45; 50; 55; 60; 65; 70; 75; 80; 85\}$. This set contains 17 elements. Therefore the number of discs that are not multiples of 5 is: $85 - 17 = 68$.
The probability of drawing a disc that is not a multiple of 5 is:
$P(\text{not } 5_m) = \frac{68}{85}$
$= 0{,}80$

(f) In part b), we worked out the probability for a disc that is a multiple of 3. Now we work out the number of elements in the set of all discs that are multiples of 4: $\{4; 8; 12; 16; 20; 24; 28; 32; 36; 40; 44; 48; 52; 56; 60; 64; 68; 72; 76; 80; 84\}$. This has 28 elements.
The probability that a disc is a multiple of either 3 or 4 is:
$P(3_m \cup 4_m) = P(3_m) + P(4_m) - P(3_m \cap 4_m)$
$= 0{,}33 + \frac{28}{85} - 0{,}33 \times \frac{28}{85}$
$= 0{,}55$

(g) The set of all discs that are a multiples of 2 and 6 is the same as the set of all discs that are a multiple of 6. Therefore the probability of drawing a disc that is both a multiple of 2 and 6 is: 0,16

(h) There is only 1 element in this set and so the probability is 0,01.

End of Chapter Exercises

1. A group of 45 children were asked if they eat Frosties and/or Strawberry Pops. 31 eat both and 6 eat only Frosties. What is the probability that a child chosen at random will eat only Strawberry Pops?
2. In a group of 42 pupils, all but 3 had a packet of chips or a Fanta or both. If 23 had a packet of chips and 7 of these also had a Fanta, what is the probability that one pupil chosen at random has:
   (a) both chips and Fanta
   (b) only Fanta
3. Use a Venn diagram to work out the following probabilities from a die being rolled:

(a) a multiple of 5 and an odd number
(b) a number that is neither a multiple of 5 nor an odd number
(c) a number which is not a multiple of 5, but is odd

4. A packet has yellow and pink sweets. The probability of taking out a pink sweet is $\frac{7}{12}$.What is the probability of taking out a yellow sweet?

5. In a car park with 300 cars, there are 190 Opels. What is the probability that the first car to leave the car park is:
   (a) an Opel
   (b) not an Opel

6. Tamara has 18 loose socks in a drawer. Eight of these are orange and two are pink. Calculate the probability that the first sock taken out at random is:
   (a) orange
   (b) not orange
   (c) pink
   (d) not pink
   (e) orange or pink
   (f) neither orange nor pink

7. A plate contains 9 shortbread cookies, 4 ginger biscuits, 11 chocolate chip cookies and 18 Jambos. If a biscuit is selected at random, what is the probability that:
   (a) it is either a ginger biscuit of a Jambo
   (b) it is not a shortbread cookie

8. 280 tickets were sold at a raffle. Ingrid bought 15 tickets. What is the probability that Ingrid:
   (a) wins the prize
   (b) does not win the prize

9. The children in a nursery school were classified by hair and eye colour. 44 had red hair and not brown eyes, 14 had brown eyes and red hair, 5 had brown eyes but not red hair and 40 did not have brown eyes or red hair.
   (a) How many children were in the school?
   (b) What is the probability that a child chosen at random has:
      i. brown eyes
      ii. red hair
   (c) A child with brown eyes is chosen randomly. What is the probability that this child will have red hair?

10. A jar has purple, blue and black sweets in it. The probability that a sweet chosen at random will be purple is $\frac{1}{7}$ and the probability that it will be black is $\frac{3}{5}$.
   (a) If I choose a sweet at random what is the probability that it will be:
      i. purple or blue
      ii. black
      iii. purple
   (b) If there are 70 sweets in the jar how many purple ones are there?
   (c) $\frac{2}{5}$ of the purple sweets in b) have streaks on them and the rest do not. How many purple sweets have streaks?

11. For each of the following, draw a Venn diagram to represent the situation and find an example to illustrate the situation.

(a) a sample space in which there are two events that are not mutually exclusive

(b) a sample space in which there are two events that are complementary

12. Use a Venn diagram to prove that the probability of either event A or B occurring is given by: (A and B are not exclusive)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

13. All the clubs are taken out of a pack of cards. The remaining cards are then shuffled and one card chosen. After being chosen, the card is replaced before the next card is chosen.

    (a) What is the sample space?

    (b) Find a set to represent the event, P, of drawing a picture card.

    (c) Find a set for the event, N, of drawing a numbered card.

    (d) Represent the above events in a Venn diagram.

    (e) What description of the sets P and N is suitable? (Hint: Find any elements of P in N and of N in P).

14. A survey was conducted at Mutende Primary School to establish how many of the 650 learners buy vetkoek and how many buy sweets during break. The following was found:

    - 50 learners bought nothing
    - 400 learners bought vetkoek
    - 300 learners bought sweets

    (a) Represent this information with a Venn diagram

    (b) If a learner is chosen randomly, calculate the probability that this learner buys:

        i. sweets only

        ii. vetkoek only

        iii. neither vetkoek nor sweets

        iv. vetkoek and sweets

        v. vetkoek or sweets

15. In a survey at Lwandani's Secondary School 80 people were questioned to find out how many read the Sowetan and how many read the Daily Sun newspaper or both. The survey revealed that 45 read the Daily Sun, 30 read the Sowetan and 10 read neither. Use a Venn diagram to find the percentage of people that read:

    (a) Only the Daily Sun

    (b) Only the Sowetan

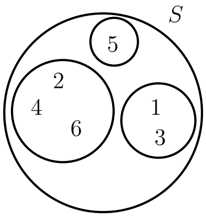
    (c) Both the Daily Sun and the Sowetan

Solutions to End of Chapter Exercises

1. 45(All) $-$ 6(only Frosties) $-$ 31(both) $=$ 8(only Strawberry Pops)
   Therefore $\frac{8}{45} = 0{,}18$

2. (a) $\frac{7}{42} = \frac{1}{6}$

   (b) Since $42-3 = 39$ had at least one, and $23+7$ had a packet of chips, then $39-30 = 9$ only had Fanta.
   $\frac{9}{42} = \frac{3}{14}$



3.

Multiples of 5 : 5

Odd number: $1, 3, 5$

Neither: $2, 4, 6$

Both: 5

- **(a)** $\frac{1}{6}$
- **(b)** $\frac{3}{6} = \frac{1}{2}$
- **(c)** $\frac{2}{6} = \frac{1}{3}$

4. $1 - \frac{7}{12} = \frac{5}{12}$

5. **(a)** $\frac{190}{300} = \frac{19}{30}$

(b) $1 - \frac{19}{30} = \frac{11}{30}$

6. **(a)** $\frac{8}{18} = \frac{4}{9}$

(b) $1 - \frac{4}{9} = \frac{5}{9}$

(c) $\frac{2}{18} = \frac{1}{9}$

(d) $1 - \frac{1}{9} = \frac{8}{9}$

(e) $\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$

(f) $1 - \frac{5}{9} = \frac{4}{9}$

7. **(a)** Total number of biscuits is $9 + 4 + 11 + 18 = 42$

$\frac{4}{42} + \frac{18}{42} = \frac{22}{42}$

$= \frac{11}{21}$

(b) $1 - \frac{9}{42} = 1 - \frac{3}{14}$

$= \frac{11}{14}$

8. **(a)** $\frac{15}{280} = \frac{3}{56}$

(b) $1 - \frac{3}{56} = \frac{53}{56}$

9. **(a)** All 4 groups are mutually exclusive, so total number of children is $44+14+5+40 = 103$.

(b) **i**. $\frac{19}{103}$

ii. $\frac{58}{103}$

(c) $\frac{14}{(14+5)} = \frac{14}{19}$

10. **(a)** **i**. Same as not black: $1 - \frac{3}{5} = \frac{2}{5}$

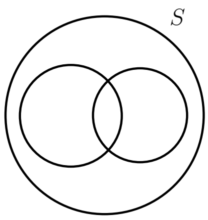
ii. $\frac{3}{5}$

iii. $\frac{1}{7}$

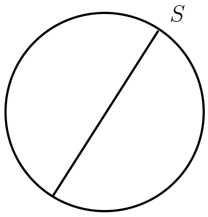
(b) $\frac{1}{7} \times 70 = 10$

(c) $10 \times \frac{2}{5} = 4$

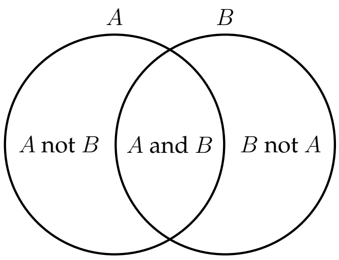
11. **(a)**



(b)



12.

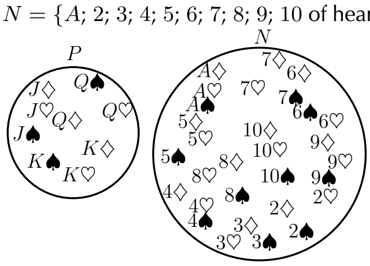


13. **(a)** {deck of cards without clubs}

(b) $P = \{J; Q; K$ of hearts, diamonds or spades$\}$

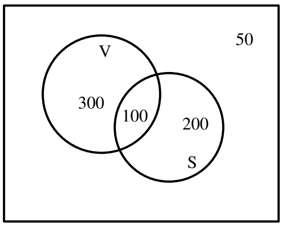
(c) $N = \{A; 2; 3; 4; 5; 6; 7; 8; 9; 10$ of hearts, diamonds or spades$\}$

(d)



(e) Mutually exclusive and complementary

14. **(a)**



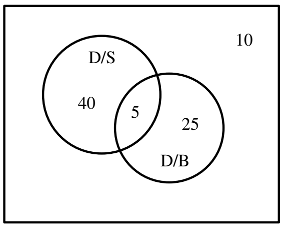
(b) (i) $\frac{200}{650} = 30{,}8\%$

(ii) $\frac{300}{650} = 46{,}2\%$

(iii) $\frac{50}{650} = 7{,}7\%$

(iv) $\frac{100}{650} = 15{,}4\%$

(v) $\frac{600}{650} = 92{,}3\%$

15. 

(a) $\frac{40}{80} = 50\%$

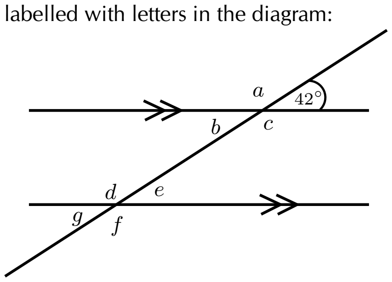
(b) $\frac{25}{80} = 31{,}25\%$

(c) $\frac{5}{80} = 6{,}25\%$

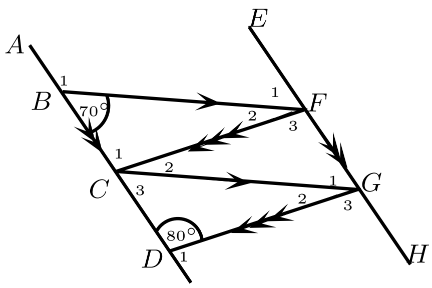
Euclidean geometry 11

Exercise 11 - 1

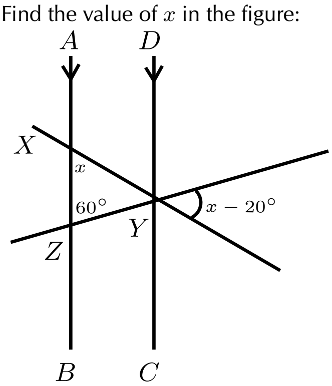
1. Use adjacent, corresponding, co-interior and alternate angles to fill in all the angles labelled with letters in the diagram:



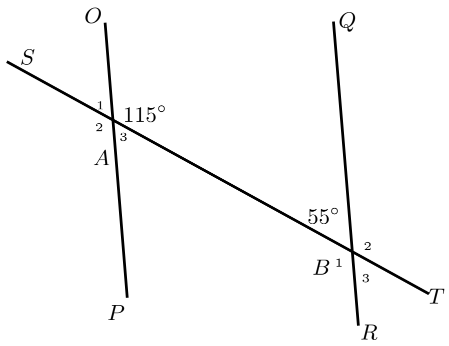
2. Find all the unknown angles in the figure:



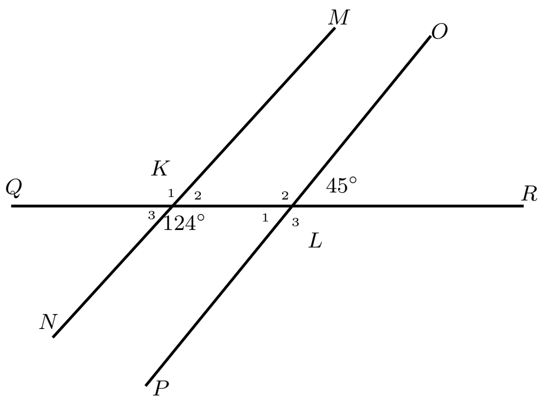
3. Find the value of x in the figure:



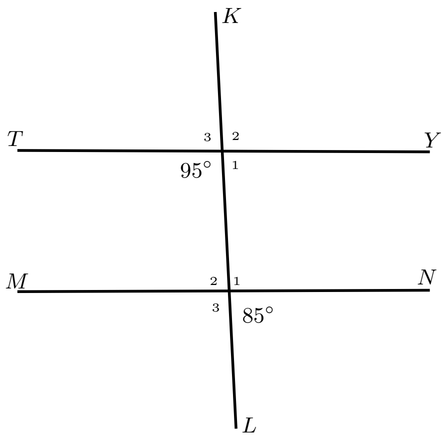
4. Determine whether the pairs of lines in the following figures are parallel:



(a)

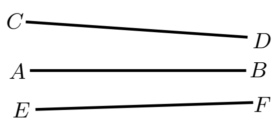


(b)



(c)

5. If AB is parallel to CD and AB is parallel to EF, explain why CD must be parallel to EF.



Solutions to Exercise 11 - 1

1. $a = 138°$
   $b = 42°$
   $c = 138°$
   $d = 138°$
   $e = 42°$
   $f = 138°$
   $g = 42°$

2. $\hat{B}_1 = 110°$
   $\hat{C}_1 = 80°$
   $\hat{C}_2 = 30°$
   $\hat{C}_3 = 70°$
   $\hat{D}_1 = 100°$
   $\hat{F}_1 = 70°$
   $\hat{F}_2 = 30°$
   $\hat{F}_3 = 80°$
   $\hat{G}_1 = 70°$
   $\hat{G}_2 = 30°$
   $\hat{G}_3 = 80°$

3. $x + 60° + x - 20° = 180°$
   $2x = 180° - 40°$
   $2x = 140°$
   $\therefore x = 70°$

4. **(a)** For interior angles $115° + 55° = 170°$
   if $\|$ the sum would be $180°$ $\therefore$ the lines are not parallel.

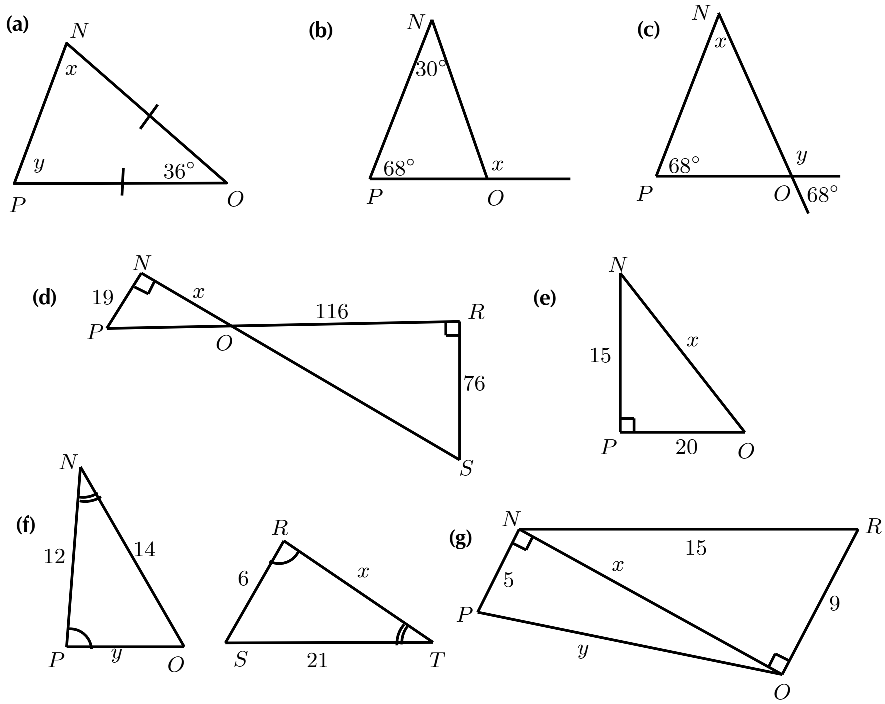
   (b) $K_2 = 180° - 124° - 56°$
   if $MN \parallel OP$ then $\hat{K}_2 = 56$ would be equal to $\hat{L} = 45°$
   $\therefore MN$ is not parallel to OP.

   (c) Let $\hat{U}$ be point of intersection of lines KL and TY and $\hat{V}$ be the point of intersection of lines KL and MN.
   $\hat{U}_4 = 96°$
   $\therefore \hat{U}_1 = 180° - 95°$ (Angles on a straight line)
   $\therefore \hat{U}_1 = 85°$
   Also, $\hat{U}_1 = 85° = \hat{V}_4$
   These are corresponding angles
   $\therefore TY \parallel MN$

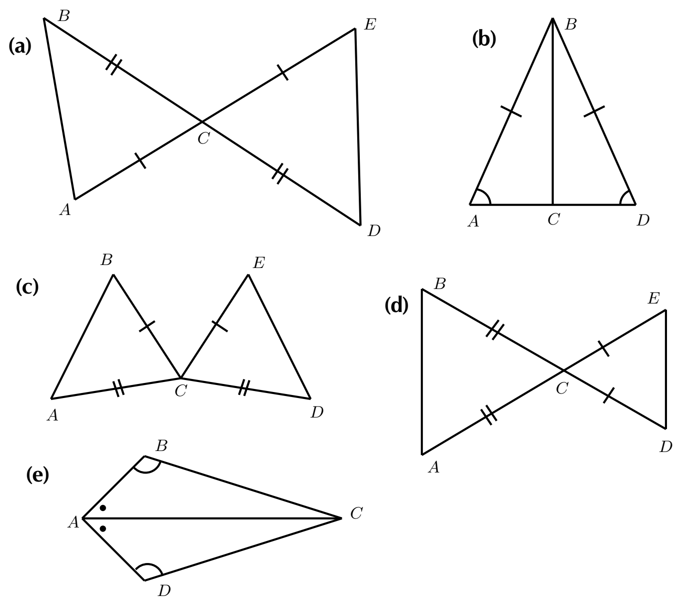
5. if $a = 2$
   and $b = a$
   the we know that $b = 2$
   Similarly if $AB \parallel CD$
   and $EF \parallel AB$
   then we know that $EF \parallel CD$

Exercise 11 - 2

1. Calculate the unknown variables in each of the following figures.



2. State whether the following pairs of triangles are congruent or not. Give reasons for your answers. If there is not enough information to make a decision, explain why.



Solutions to Exercise 11 - 2

1. **(a)** The triangle is isosceles,
   $\therefore x = y$
   $180° = 36° - 2x$
   $2x = 144°$
   $\therefore x = y = 72°$

   (b) x is an exterior angle
   $\therefore x = 30° + 68°$
   $= 98°$

   (c) y is an exterior angle
   $\therefore y = x + 68° = 112°$
   $x = 112° - 68° = 44°$

   (d) Interior angles of a triangle add up to $180°$
   $\therefore \hat{P} = 180° - \hat{N} - N\hat{O}P$
   and $\hat{S} = 180° - \hat{R} - R\hat{O}S$
   $\hat{N} = \hat{R} = 90°$ and $N\hat{O}P = R\hat{O}P$ (opposite angles)
   $\hat{P} = \hat{S}$
   Therefore $\triangle NPO$ and $\triangle ROS$ are similar because they have the same angles.
   Similar triangles have proportional sides
   $\therefore \frac{NP}{RS} = \frac{NO}{OR}$
   $\frac{19}{76} = \frac{x}{116}$
   $\therefore x = 19$ units

   (e) From the theorem of Pythagoras we have
   $x^2 = 15^2 + 20^2$

$\therefore x = \sqrt{625}$
$= 25$ units

(f) $\triangle NPO ||| \triangle TSR$ (AAA)
$\therefore \frac{OP}{NP} = \frac{SR}{TR}$
$\frac{y}{12} = \frac{6}{x}$
$\therefore xy = 72$
and $\frac{NO}{OP} = \frac{TS}{TR}$
$\frac{14}{12} = \frac{21}{x}$
$\therefore x = \frac{21 \times 12}{14}$
$\therefore x = 18$ units
$\therefore y(18) = 72$
$\therefore y = 4$ units

(g) From the theorem of Pythagoras:
$x^2 = 15^2 - 9^2$
$x = \sqrt{144}x = 12$ units
And
$y^2 = x^2 + 5^2$
$y^2 = 144 + 25$
$y = \sqrt{169}$
$y = 13$ units

2. **(a)** $\triangle ABC \equiv \triangle EDC$ (SAS: $BC = CD$, $EC = AC$ and $B\hat{C}A = E\hat{C}D$)

(b) We have two equal sides ($AB = BD$ and BC is common to both triangles) and one equal angle ($\hat{A} = \hat{D}$) but the sides do not include the known angle. The triangles therefore do not have a SAS and are therefore not congruent. (Note: $A\hat{C}B$ is not necessarily equal to $D\hat{C}B$ because it is not given that $BC \perp AD$)

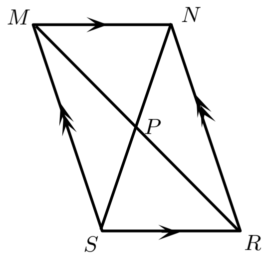
(c) There is not enough information given. We need at least three facts about the triangles and in this example we only know two sides in each triangle.

(d) There is not enough information given. Although we can work out which angles are equal we are not given any sides as equal. All we know is that we have two isosceles triangles. Note how this question differs from part a). In part a) we were given equal sides in both triangles, in this question we are only given that sides in the same triangle are equal.

(e) $\triangle ABD ||| \triangle ADC$ (AAS: CA is a common side, and two angles are given as being equal.)

Exercise 11 - 3

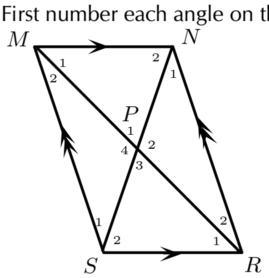
1. Prove that the diagonals of the parallelogram $MNRS$ bisect one another at P.



Hint: Use congruency.

Solutions to Exercise 11 - 3

1. First number each angle on the given diagram:



In $\triangle MNP$ and $\triangle RSP$

$\hat{M}_1 = \hat{R}_1$ (alt $\angle$'s)

$\hat{P}_1 = \hat{P}_3$ ($MN \parallel SR$, vert. opp. $\angle$'s)

$MN = RS$ (opp. sides of parallelogram)

$\therefore \triangle MNP ||| \triangle RSP$ (AAS)

$\therefore MP = RP$

$\therefore P$ is the mid-point of MR

Similarly, in $\triangle MSP$ and $\triangle RNP$

$\hat{M}_2 = \hat{R}_2$ (alt $\angle$'s)

$\hat{P}_4 = \hat{P}_2$ ($MN \parallel SR$, vert. opp. $\angle$'s)

$MS = RN$ (opp. sides of parallelogram)

$\therefore \triangle MSP ||| \triangle RNP$ (AAS)

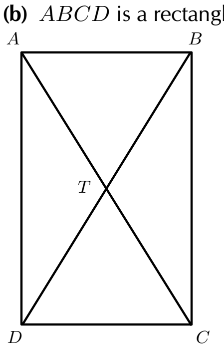
$\therefore SP = NP$

$\therefore P$ is the mid-point of NS

Therefore the diagonals bisect each other at point P.

Exercise 11 - 4

1. $ABCD$ is a quadrilateral. Diagonals AC and BD intersect at T. $AC = BD$, $AT = TC$, $DT = TB$. Prove that:
   (a) $ABCD$ is a parallelogram.
   (b) $ABCD$ is a rectangle.

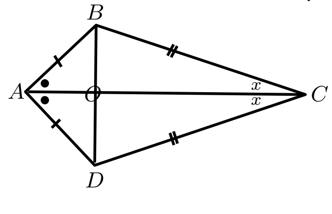


Solutions to Exercise 11 - 4

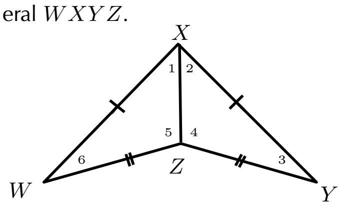
1. **(a)** $AT = TC$ (given)
   $\therefore DB$ bisects AC at T
   and $DT = TB$ (given)
   $\therefore AC$ bisects DB at T
   therefore quadrilateral $ABCD$ is a parallelogram (diagonals bisect each other)
   (b) $AC = BD$ (given)
   Therefore $ABCD$ is a rectangle (diagonals are of equal length)

Exercise 11 - 5

1. Use the sketch of kite $ABCD$ to prove the diagonals are perpendicular to one another.

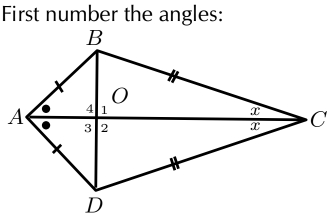


2. Explain why quadrilateral $WXYZ$ is a kite. Write down all the properties of quadrilateral $WXYZ$.

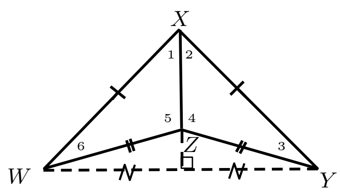


Solutions to Exercise 11 - 5

1. First number the angles:



In $\triangle ADB$
let $\hat{A}_1 = \hat{A}_2 = t$
and let $\hat{D} = \hat{B} = p$
$\therefore$ in $\triangle ADB$
$2t + 2p = 180°$
$\therefore t + p = 90°$
But $\hat{O}_1 = \hat{B} + \hat{A}_1$ (ext. $\angle =$ sum of two opp. int. $\angle$'s)
$\hat{O}_1 = p + t$
$= 90°$
$\therefore AC \perp BD$

2. 

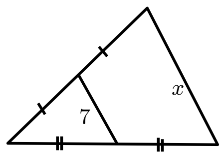
Quadrilateral $WXYZ$ is a kite because is has two pairs of adjacent sides that are equal in length.

- $\hat{W}_1 = \hat{Y}_1$ (opp. angles equal)
- Diagonal between equal sides bisects the other diagonal: $WP + PY$
- $\hat{X}_1 = \hat{X}_2$ (diagonal bisects int. $\angle$)
  $\hat{Y}_1 = \hat{Y}_2$ (diagonal bisects int. $\angle$)
- $WY \perp PX$

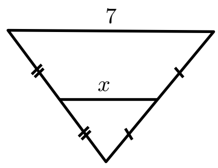
Exercise 11 - 6

1. Find x and y in each of the following:

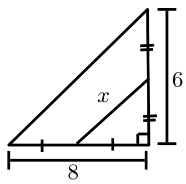
(a)



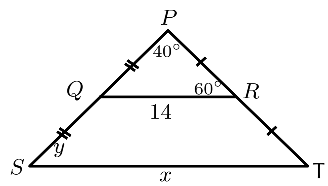
(b)



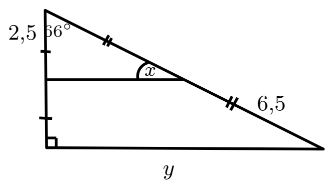
(c)



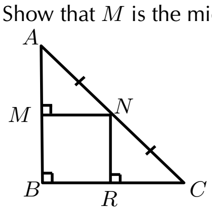
(d)



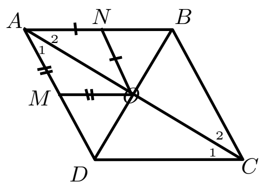
(e)



2. Show that M is the mid-point of AB and that $MN = RC$.



3. $ABCD$ is a rhombus with $AM = MO$ and $AN = NO$. Prove $ANOM$ is also a rhombus.



Solutions to Exercise 11 - 6

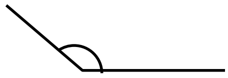
1. **(a)** $x = 14$ units
   (b) $x = 3{,}5$ units
   (c) $x = 5$ units
   (d) $x = 28$ units; $y = 80°$
   (e) $x = 24°$;$y = 12$ units
2. N is the mid-point of AC (given: $AN = NC$)
   and $\hat{B} = \hat{M} = 90°$ (given)
   But these are equal, corresponding angles
   $MN \parallel BR$
   $BM \parallel RN$
   $\therefore M$ is the mid-point of line AB

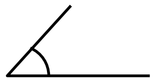
   $\therefore MN = \frac{1}{2}BC$ (mid-point theorem)
   but $MN = BR$ (opp. sides of parallelogram $MNRB$)
   and $BL = BR + RC$
   $\therefore MN = \frac{1}{2}(BR + RC)$
   $2MN = MN + RC$
   $\therefore MN = RC$
3. In $\triangle AMO$ and $\triangle ANO$
   $\hat{A}_1 = \hat{A}_2$ (given rhombus $ABCD$, diagonal AC bisects $\hat{A}$)
   $\therefore \hat{A}_1 = \hat{O}_1$ ($\angle$'s opp. equal sides, given $AM = MO$ and $AN = NO$)
   and $\hat{A}_2 = \hat{O}_2$
   $\therefore \hat{A}_2 = \hat{O}_1$ and $\hat{A}_1 = \hat{O}_2$
   but these are alt. $\angle$'s
   $\therefore AN \parallel MO$ and $AM \parallel NO$
   $\therefore AMON$ is a parallelogram
   $\therefore AM = NO$ (opp. sides parm equal)
   $AM = MO = ON = NO$
   $\therefore AMON$ is a rhombus (all sides equal and parallel)

End of Chapter Exercises

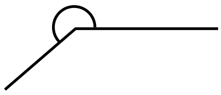
1. Identify the types of angles shown below:

(a) 

(b) 

(c) 

(d) 

(e) 

(f) An angle of $91°$

(g) An angle of $180°$

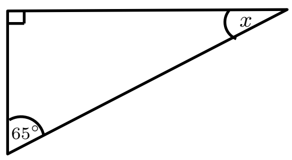
(h) An angle of $210°$

2. Assess whether the following statements are true or false. If the statement is false, explain why:

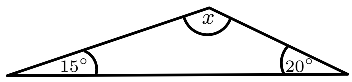
(a) A trapezium is a quadrilateral with two pairs of opposite sides that are parallel.
(b) Both diagonals of a parallelogram bisect each other.
(c) A rectangle is a parallelogram that has one corner angles equal to $90°$.
(d) Two adjacent sides of a rhombus have different lengths.
(e) The diagonals of a kite intersect at right angles.
(f) All squares are parallelograms.
(g) A rhombus is a kite with a pair of equal, opposite sides.
(h) The diagonals of a parallelogram are axes of symmetry.
(i) The diagonals of a rhombus are equal in length.
(j) Both diagonals of a kite bisect the interior angles.

3. Calculate the size of the third angle (x) in each of the diagrams below:

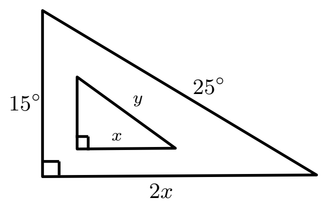
(a)



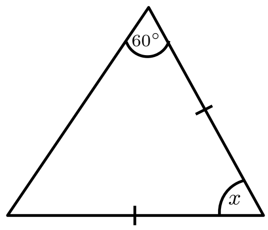
(b)



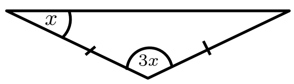
(c)



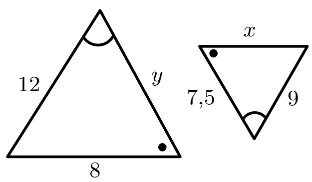
(d)



(e)

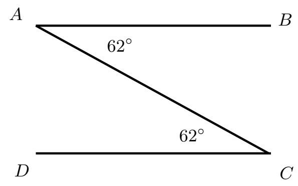


(f)

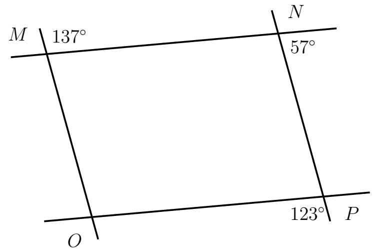


4. Find all the pairs of parallel lines in the following figures, giving reasons in each case.

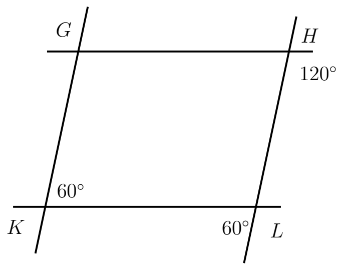
(a)



(b)

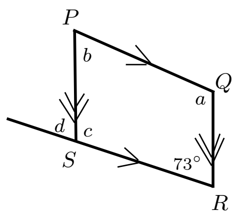


(c)

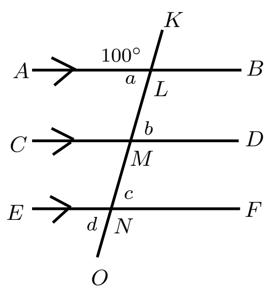


5. Find angles a, b, c and d in each case, giving reasons:

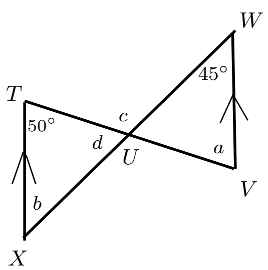
(a)



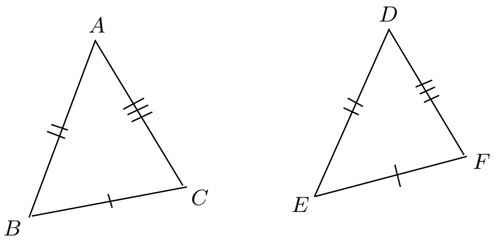
(b)

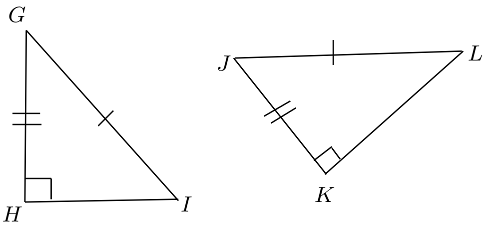


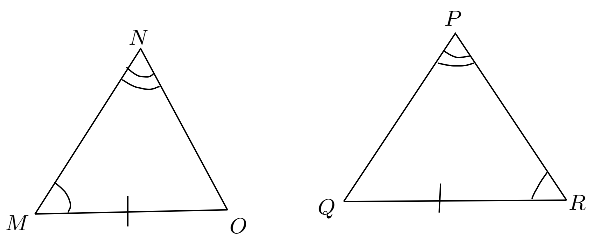
(c)

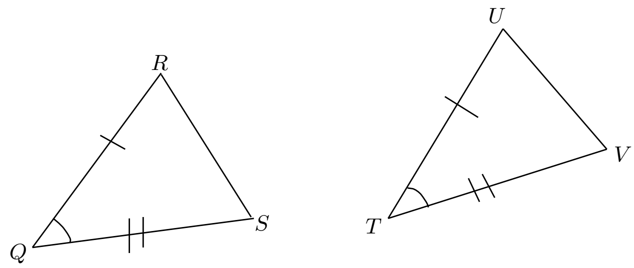


6. Say which of the following pairs of triangles are congruent with reasons.

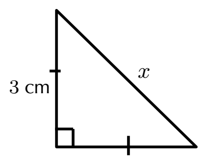
(a)


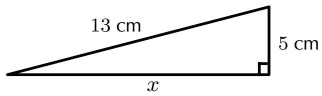
(b)


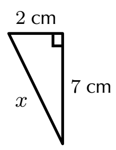
(c)


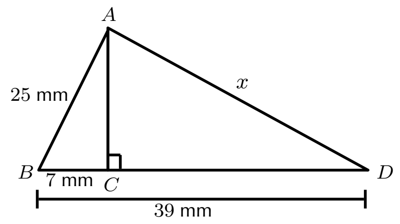
(d)


7. Using Pythagoras' theorem for right-angled triangles, calculate the length x:

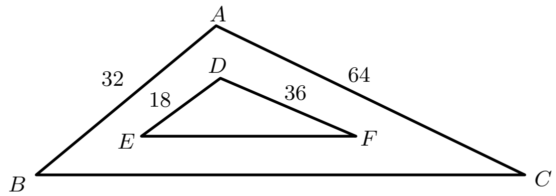
(a)


(b)


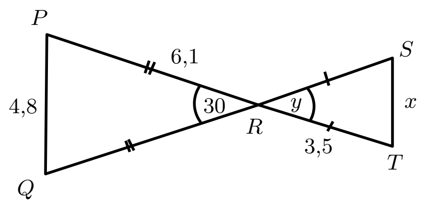
(c)


(d) 

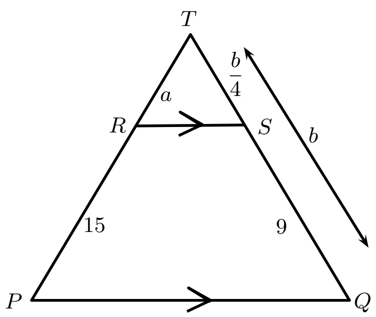
8. Consider the diagram below. Is $\triangle ABC ||| \triangle DEF$? Give reasons for your answer.



9. Explain why $\triangle PQR$ is similar to $\triangle TRS$ and calculate the values of x and y.



10. Calculate a and b:

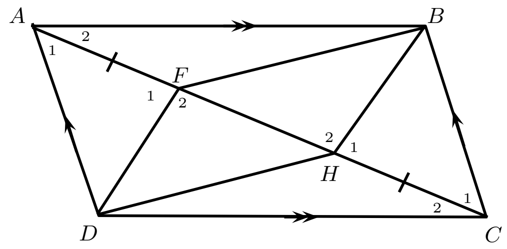


11. $ABCD$ is a parallelogram with diagonal AC.
Given that $AF = HC$, show that:

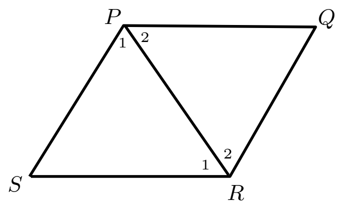
(a) $\triangle AFD \equiv \triangle CHB$

(b) $DF \parallel HB$

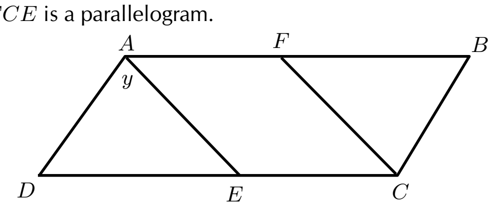
(c) $DFBH$ is a parallelogram



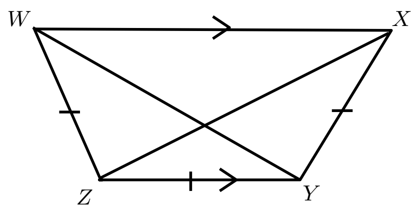
12. $\triangle PQR$ and $\triangle PSR$ are equilateral triangles. Prove that $PQRS$ is a rhombus:



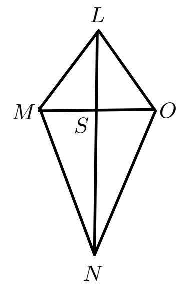
13. Given parallelogram $ABCD$ with AE and FC, AE bisecting $\hat{A}$ and FC bisecting $\hat{C}$:
    - **(a)** Write all interior angles in terms of y.
    - **(b)** Prove that $AFCE$ is a parallelogram.



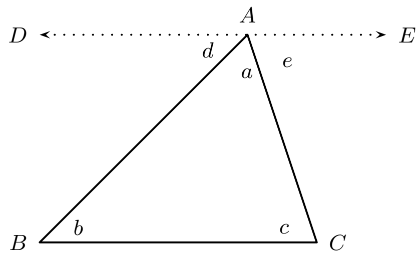
14. Given that $WZ = ZY = YX$, $\hat{W} = \hat{X}$ and $WZ \parallel ZY$, prove that:
    - **(a)** XZ bisects $\hat{X}$
    - **(b)** $WY = XZ$



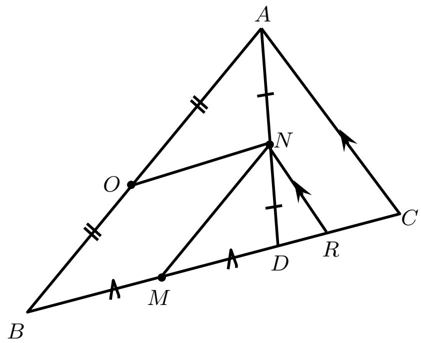
15. $LMNO$ is a quadrilateral with $LM = LO$ and diagonals that intersect at S such that $MS = SO$. Prove that:
    - **(a)** $M\hat{L}S = S\hat{L}O$
    - **(b)** $\triangle LON \equiv \triangle LMN$
    - **(c)** $MO \perp LN$



16. Using the figure below, show that the sum of the three angles in a triangle is 180°. Line DE is parallel to BC.



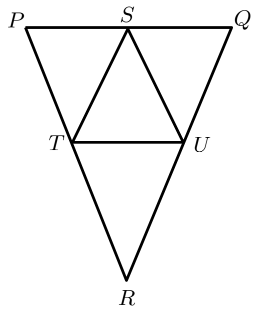
17. D is a point on BC, in $\triangle ABC$. N is the mid-point of AD. O is the mid-point of AB and M is the mid-point of BD. $NR\, parallel AC$.



(a) Prove that $OBMN$ is a parallelogram.

(b) Prove that $BC = 2MR$.

18. PQR is an isoceles with $PR = QR$. S is the mid-point of PQ, T is the mid-point of PR and U is the mid-point of RQ.

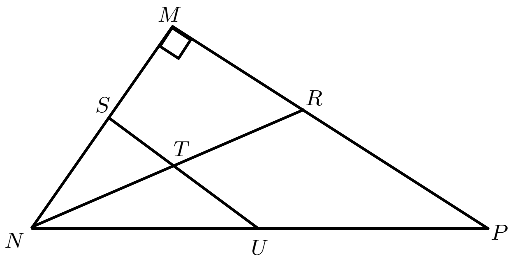


(a) Prove $\triangle STU$ is also isoceles.

(b) What type of quadrilateral is $STRU$? Motivate your answer.

(c) If $R\hat{T}U = 68°$ calculate, with reasons, the size of $T\hat{S}U$.

19. In $\triangle MNP$, $M = 90°$, S is the mid-point of MN and T is the mid-point of NR.



(a) Prove M is the mid-point of NP.

(b) If $ST = 4$ cm and the area of $\triangle SNT$ is 6 cm^2, calculate the area of $\triangle MNR$.

(c) Prove that the area of $\triangle MNR$ will always be four times the area of $\triangle SNT$, let $ST = x$ units and $SN = y$ units.

Solutions to End of Chapter Exercises

1. **(a)** Straight angle
(b) Obtuse angle
(c) Acute angle
(d) Right angle
(e) Reflex angle
(f) Obtuse angle
(g) Straight angle
(h) Reflex angle

2. **(a)** False - a trapezium only has one pair of opposite parallel sides.
(b) True.
(c) True.
(d) False - two adjacent sides of a rhombus are equal in length.
(e) True.
(f) True.
(g) True.

(h) False - the diagonals of a rhombus are not equal in length.

(i) False - only one diagonal of a kite bisects two pairs of interior angles.

3. **(a)** $x = 180° - 90° - 65° = 25°$

(b) $x = 180° - 20° - 15° = 145°$

(c) $25^2 = 15^2 + (2x)^2$

$4x^2 = 400$

$x^2 = 100$

$\therefore x = 10$ units

$\frac{x}{2x} = \frac{y}{25}$

$\therefore y = 12{,}5$ units

(d) $x = 60°$

(e) $x + x + 3x = 180°$

$\therefore 5x = 180°$

$\therefore x = 36°$

(f) $\frac{x}{9} = \frac{8}{12}$

$\therefore x = 6$ units

$\frac{y}{12} = \frac{7.5}{9}$

$\therefore y = 10$ units

4. **(a)** $AB \parallel CD$ (alt. int. $\angle$'s equal)

(b) NP not $\parallel$ MO (corresp. $\angle$'s not equal)

$MN \parallel OP$ (corresp. $\angle$'s equal)

(c) $GH \parallel HL$ (corresp. $\angle$'s equal)

5. **(a)** $a = 180° - 73° = 107°$ (co-interior $\angle$'s)

$b = 180° - 107° = 73°$ (co-interior $\angle$'s)

$c = 180° - 73° = 107°$ ($\angle$'s on a str. line)

$d = 73°$ (corresp. $\angle$'s)

(b) $a = 80°$ ($\angle$'s on a str. line)

$b = 80°$ (alt. int. $\angle$'s)

$c = 80^{-73\circ}$ (corresp. $\angle$'s)

$d = 80°$ (opp. $\angle$'s)

(c) $a = 50°$ (alt. int. $\angle$'s)

$b = 45°$ (alt. int. $\angle$'s)

$c = 95°$ (sum of int. $\angle$'s)

$d = 85°$ (sum of $\angle$'s in a $\triangle$)

6. **(a)** Congruent by SSS

(b) Congruent by RHS

(c) Congruent by AAS

(d) Congruent by SAS

7. **(a)** $x = \sqrt{3^2 + 3^2} = \sqrt{9 + 9} = \sqrt{18} = 4{,}24$ cm

(b) $x = \sqrt{13^2 + 5^2} = \sqrt{139 - 25} = \sqrt{144} = 12$ cm

(c) $x = \sqrt{2^2 + 7^2} = \sqrt{4 + 49} = \sqrt{53} = 7{,}28$ cm

(d) $AC = \sqrt{25^2 - 7^2} = 576$

$\therefore AC = 24$

$x^2 = 32^2 + 24^2$

$\therefore x = 40$ mm

8. $\frac{ED}{BA} = \frac{18}{32} = \frac{9}{16}$

$\frac{EF}{BC} = \frac{32}{64} = \frac{9}{16}$
These pairs of sides are in proportion,
$\therefore \triangle ABC ||| \triangle DEF$

9. $y = 30°$ (vert. opp. $\angle$'s)
$\frac{x}{4{,}8} = \frac{3{,}5}{6{,}1}$
$\therefore x = 2{,}75$
$\hat{P} = \hat{Q}$ (equal $\angle$opp. sides isoceles $\triangle$)
and $\hat{S} = \hat{T}$ (equal $\angle$opp. sides isoceles $\triangle$)
$\therefore \triangle PQR ||| \triangle TRS$ (AAA)

10. $\frac{a}{a+15} = \frac{\frac{b}{4}}{b}$
$a = (a+15)\left(\frac{1}{4}\right)$
$4a = a + 15$
$3a = 15$
$\therefore a = 5$

$b = \frac{b}{4} + 9$
$4b = b + 36$
$3b = 36$
$\therefore b = 12$

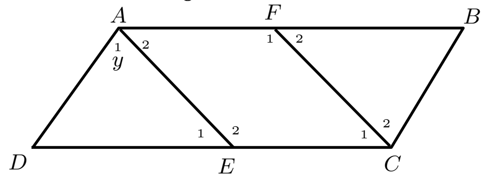
11. **(a)** $\hat{A}_1 = \hat{C}_1$ (alt. $\angle$'s, $AD \parallel BC$)
$AD = BC$ (opp. sides of parallelogram equal)
$AF = HC$ (given)
$\therefore \triangle AFD \equiv \triangle CHB$ (SAS)

(b) $\hat{F}_1 = \hat{H}_1$ ($\triangle AFD \equiv \triangle CHB$)
$\therefore \hat{F}_1 + \hat{F}_2 = 180°$ ($\angle$'s on str. line)
and $\hat{H}_1 + \hat{H}_2 = 180°$ ($\angle$'s on str. line)
$\therefore \hat{F}_2 + \hat{H}_2$
but these are alternate $\angle$'s,
$\therefore DF \parallel HB$

(c) $FD = HB$ ($\triangle AFD \equiv \triangle CHB$)
and $DF \parallel HB$ (proved above)
$\therefore DFBH$ is a parallelogram (one pair opp. sides equal parallel)

12. Given $\triangle PSR \equiv \triangle PQR$, with common side PR
$PS = SR = PR = QR$
$\therefore$ all sides equal in length
$\therefore PQRS$ is a rhombus

13. First number the angles:



(a) $\hat{A}_2 = y$ (given AE bisects $\hat{A}$)
$\hat{E}_1 = y$ (alt. $\angle$'s, $AB \parallel DC$ in parm $ABCD$)

$\therefore hatE_2 = 180° - y$ ($\angle$'s on str. line)

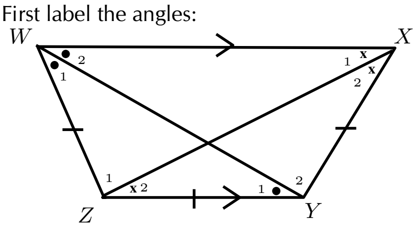
$\hat{C}_1 = \hat{C}_2$ (given FC bisects $\hat{C}$)
and $\hat{A} = \hat{C}$ (opp. $\angle$'s parm $ABCD$ equal)
$\therefore \hat{C}_1 = \hat{C}_2 = y$

$\therefore \hat{F}_2 = \hat{C}_1 = y$ (alt. $\angle$'s, $AB \parallel DC$)
$\therefore \hat{F}_1 = 180° - y$

In $\triangle ADE$
$\hat{D} + \hat{A}_1 + \hat{E}_1 = 180°$ (sum of $\angle$'s in $\triangle$)
$\therefore \hat{D} + y + y = 180°$
$\therefore \hat{D} = 180° - 2y$
$\hat{D} = 90° - y$
$\therefore \hat{B} = 90° - y$ (opp. $\angle$'s parm $ABCD$ equal)

(b) $AF \parallel EC$ (opp. sides parm $ABCD$ equal)
and $\hat{C}_1 + \hat{E}_2 = y + (180° - y)$
$\therefore$ the sum of the co-interior angles is $180°$
$\therefore AE \parallel FC$
$\therefore AFCE$ is a parallelogram (both opp. sides parallel)

14. First label the angles:



(a) IN $\triangle XYZ$
$\hat{X}_2 = \hat{Z}_2$ ($\angle$'s opp. equal sides of isoceles $\triangle$)
and $\hat{X}_1 = \hat{Z}_2$ (alt. $\angle$'s, $WX \parallel ZY$)
$\therefore \hat{X}_1 = \hat{X}_2$
$\therefore XZ$ bisects $\hat{X}$

(b) Similarly, WY bisects $\hat{W}$
$\therefore \hat{W}_1 = \hat{W}_2$
and $\hat{W} = \hat{X}$ (given)
$\therefore \hat{W}_1 = \hat{W}_2 = \hat{X}_1 = \hat{X}_2$
and $\hat{W}_1 = \hat{Y}_1$ ($\angle$'s opp. equal sides)
In $\triangle WZY$ and $\triangle XYZ$
$WZ = XY$ (given)
ZY is a common side
$\hat{Z} = \hat{Y}$(third $\angle$ in $\triangle$)
$\therefore \triangle WZY \equiv \triangle XYZ$ (SAS)

15. **(a)** In $\triangle LMS$ and $\triangle LOS$
$LM = LO$ (given)
$MS = SO$ (given)

LS is a common side
$\therefore \triangle LMS \equiv \triangle LOS$ (SSS)
$\therefore \hat{L_1} = \hat{L_2}$

(b) In $\triangle LON$ and $\triangle LMN$
$LO = LM$ (given)
$\hat{L_1} = \hat{L_2}$ (proved above)
LN is a common side
$\therefore \triangle LON \equiv \triangle LMN$ (SSS)
$\therefore \hat{L_1} = \hat{L_2}$

(c) In $\hat{M_1} = \hat{O_1}$ ($\triangle LON \equiv \triangle LMN$)
and $\hat{L_1} = \hat{L_2}$ (proved above)
$\therefore$ in $\triangle LMO$
$\hat{L_1} + \hat{L_2} + \hat{M_1} + \hat{O_1} = 180°$ (sum of $\angle$'s in $\triangle$)
$\therefore 2\hat{L_1} + 2\hat{O_1} = 180°$
$2(\hat{L_1} + \hat{O_1}) = 180°$
$\hat{L_1} + \hat{O_1} = 90°$
but $\hat{S_1} = \hat{O_1} + \hat{L_2}$ (ext. $\angle$ of $\triangle =$ sum of int. opp. $\angle$'s)
$\therefore \hat{S_1} = 90°$
$\therefore MO \perp LN$

16. $DE \parallel BC$
$e = c$ (alt. int. $\angle$'s)
$d = b$ (alt. int. $\angle$'s)
We know that $d + a + e = 180°$
And we have shown that $e = c$ and $d = e$ therefore we can replace d and e in the diagram to get:
$a + b + c = 180°$
Therefore the angles in a triangle do add up to $180°$.

17. **(a)** $AO = OB$ (given)
$AN = ND$ (given)
$\therefore ON \parallel BD$ (mid-pt theorem)
$BM = MD$ (given)
$AN = ND$ (given)
$\therefore MN \parallel AB$
$\therefore OBMN$ is a parallelogram (opp. sides parallel)

(b) $AN = NC$ (given)
$NR \parallel AC$ (given)
$\therefore DR = RC$ (mid-pt theorem)
$\therefore DR = \frac{1}{2}DC$
$MD = \frac{1}{2}BD$ (given)
$\therefore MD + DR = \frac{1}{2}(BD + DC)$
$MR = \frac{1}{2}BC$
$\therefore BC = 2MR$

18. **(a)** $PT = \frac{1}{2}PR$ (given)
S mid-point of PQ
U mid-point of RQ
$SU = \frac{1}{2}PR$
$\therefore SU = PT$

S mid-point of PQ
T mid-point of PR
$\therefore ST = \frac{1}{2}QR = QU$
But $PR = QR$ (given)
$\therefore SU = ST$
$\therefore \triangle STU$ is isoceles.

(b) $STRU$ is a rhombus. It is a parallelogram ($SU \parallel TR$ and $ST \parallel UR$) with four equal sides ($US = ST = TR = RU$).

(c) $R\hat{T}U = 68°$
$\therefore T\hat{U}S = 68°$ (alt $\angle$'s, $TR \parallel SU$)
$\therefore S\hat{T}U = 68°$ ($SU = ST$)
$\therefore T\hat{S}U = 180° - 136°$
$= 44°$

19. **(a)** $NS = SM$ (given)
$NT = TR$ (given)
$\therefore ST \parallel MR$ (mid-pt theorem)

(b) $N\hat{S}T = 90°$
$\therefore$ Area $\triangle SNT = \frac{1}{2}ST(SN)$ (corresp. $\angle$'s, $ST \parallel MR$)
$6 = \frac{1}{2}(4)SN$
$\therefore SN = 3$ cm
$\therefore MN = 6$ cm
$MR = 2ST = 8$ cm
Area $\triangle MNR = \frac{1}{2}MR \times MN$
$= \frac{1}{2}(8)(6)$
$= 24$ cm^2

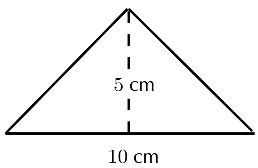
(c) Let ST be x units
$\therefore MR$ will be $2x$
Let SN be y units
$\therefore$MN will be $2y$
Area $\triangle SNT = \frac{1}{2}xy$
$\frac{1}{2}xy$
Area $\triangle MNR = \frac{1}{2}(2x)(2y)$
$= 2xy$
$\therefore$ Area $\triangle MNR = 4(\frac{1}{2}xy)$
$= 4$(Area $\triangle SNT$)

Measurements 12

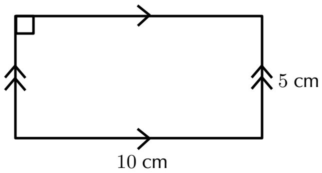
Exercise 12 - 1

Find the areas of each of the polygons below:

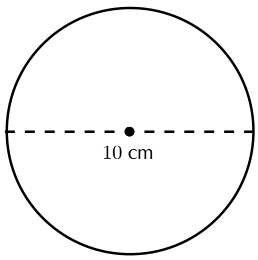
1.



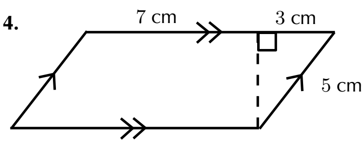
2.



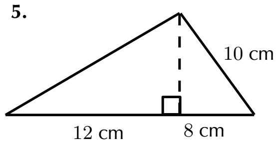
3.



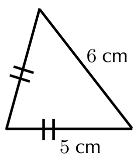
4.



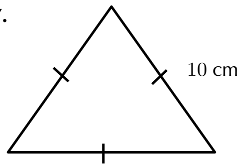
5.



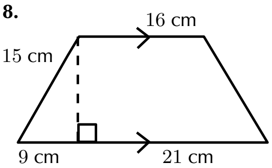
6.



7.



8.



Solutions to Exercise 12 - 1

1. $A = \frac{1}{2}\text{base} \times \text{height}$
   $A = \frac{1}{2}(10)(5)$
   $A = 25\text{ cm}^2$
2. $A = \text{length} \times \text{breadth}$
   $A = (10)(5)$
   $A = 50\text{ cm}^2$
3. $A = \pi r^2$
   $A = 3{,}14159(5^2)$

$A = 78{,}5398$ cm^2
$A \approx 79$ cm^2
Note that the radius is half the diameter.

4. $A =$ base $\times$ perpendicular height
$A = (10)(\sqrt{5^2 - 3^2})$
$A = (10)(4)$
$A = 40$ cm^2

5. $h^2 = 10^2 + 8^2$
$h = \sqrt{164}$
$h = 12{,}81$ cm
$A = \frac{1}{2}$base $\times$ height
$A = \frac{1}{2}(12{,}81)(20)$
$A \approx 128$ cm^2

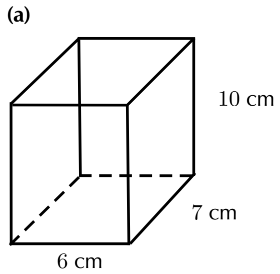
6. We first need to construct the perpendicular height. If we do this such that we divide th base in half we get:
$h^2 = 3^2 + 5^2$
$h = \sqrt{34}$
$h = 5{,}83$ cm
$A = \frac{1}{2}$base $\times$ height
$A = \frac{1}{2}(6)(5{,}83)$
$A = 17{,}5$ cm^2

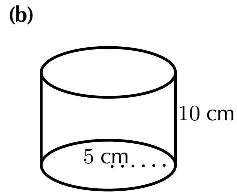
7. Once again we construct the perpendicular height. If we do this such that we divide the base in half we get:
$h^2 = 10^2 + 5^2$
$h = \sqrt{125}$
$h = 11{,}18$ cm
$A = \frac{1}{2}$base $\times$ height
$A = \frac{1}{2}(10)(11{,}18)$
$A = 60$ cm^2

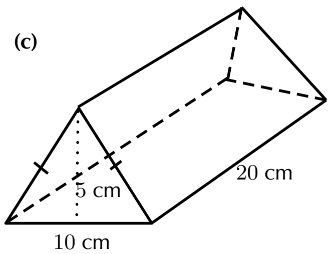
8. $h^2 = 15^2 + 9^2$
$h = \sqrt{306}$
$h = 17{,}49$ cm
$A =$ base $\times$ height
$A = (30)(17{,}49)$
$A = 525$ cm^2

Exercise 12 - 2

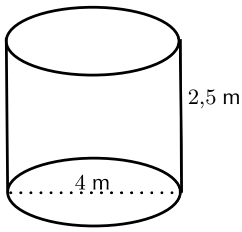
1. Calculate the surface area of the following prisms:







2. If a litre of paint covers an area of 2 m^2, how much paint does a painter need to cover:
   (a) a rectangular swimming pool with dimensions 4 m $\times$ 3 m $\times$ $2{,}5$ m (the inside walls and floor only).
   (b) the inside walls and floor of a circular reservoir with diameter 4 m and height $2{,}5$ m.



Solutions to Exercise 12 - 2

1. **(a)** Area large rectangle = perimeter of small rectangle $\times$ length
$= (10 + 7 + 10 + 7) \times 6$
$= 34 \times 6$
$= 204$ cm^2
Area of 2 $\times$small rectangle $= 2(7 \times 10)$
$= 2(70)$
$= 140$ cm^2
Area of large rectangle $+\ 2\ \times$(small rectangle) $= 204 + 140 = 344\ cm^2$

(b) Area of large rectangle $=$ circumference of circle $\times$ length
$= 2\pi \times r \times l$
$= 2\pi \times (5) \times 10$
$= 314{,}16$ cm^2
Area of circle $= \pi r^2$
$= \pi 5^2$
$= 78{,}5$ cm^2
Surface area $=$ area large rectangle $+$ 2(area of circle)
$= 314{,}16 + 2(78{,}5)$
$= 471{,}2$ cm^2

(c) $x^2 = 5^2 + (\frac{10}{2})^2$
$x^2 = 5^2 + 5^2$
$= 25 + 25$
$x = \sqrt{50}$
$x = 7{,}07$ cm
Perimeter of triangle $= 10 + 7{,}07 + 7{,}07$
$= 24{,}14$ cm
Area of large rectangle $=$ perimeter of triangle $\times$ length
$= 24{,}14 \times 20$
$= 482{,}8$ cm^2
Area of triangle $= \frac{1}{2} b \times h$
$= \frac{1}{2} \times 5 \times 10$
$= 25$ cm^2
Surface area $=$ area large rectangle $+$ 2(area of triangle)
$= 482{,}8 + 2(25) = 532{,}8$ cm^2

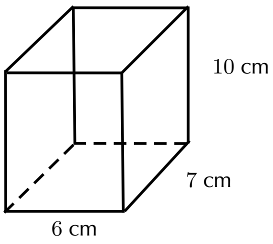
2. **(a)** Surface area $=$ area of bottom of pool$+2$(area of long sides)$+2$(area of short sides)
$= (4 \times 3) + 2(4 \times 2{,}5) + 2(3 \times 2{,}5)$
$= 12 + 20 + 15$
$= 47$ m^2
The painter will need $\frac{47}{2} = 24$ litres of paint (rounded up to the nearest litre).

(b) We first need to work out r:
$r = \frac{4}{2} = 2$ m
Surface area $=$ area of bottom of reservoir $+$ area of inside of reservoir
$= (\pi r^2) +$ (circumference of base $\times$ height of reservoir)
$= (\pi(2)^2) + (2(\pi)(2) \times 2{,}5)$
$= 43{,}98$ m
The painter will need $\frac{44}{2} = 22$ litres of paint (rounded up to the nearest litre.

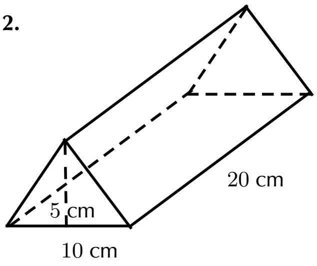
Exercise 12 - 3

Calculate the volumes of the following prisms (correct to one decimal place):

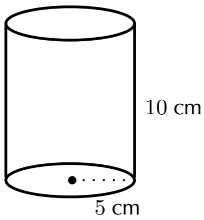
1.



2.



3.

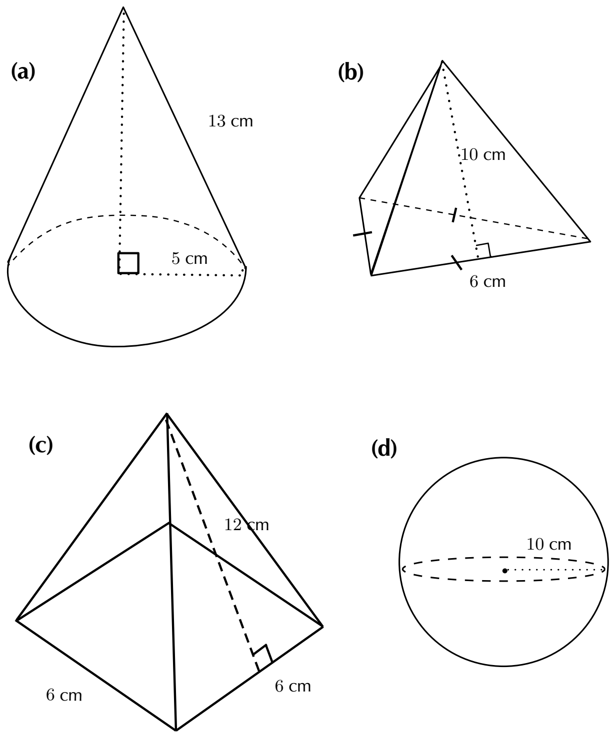


Solutions to Exercise 12 - 3

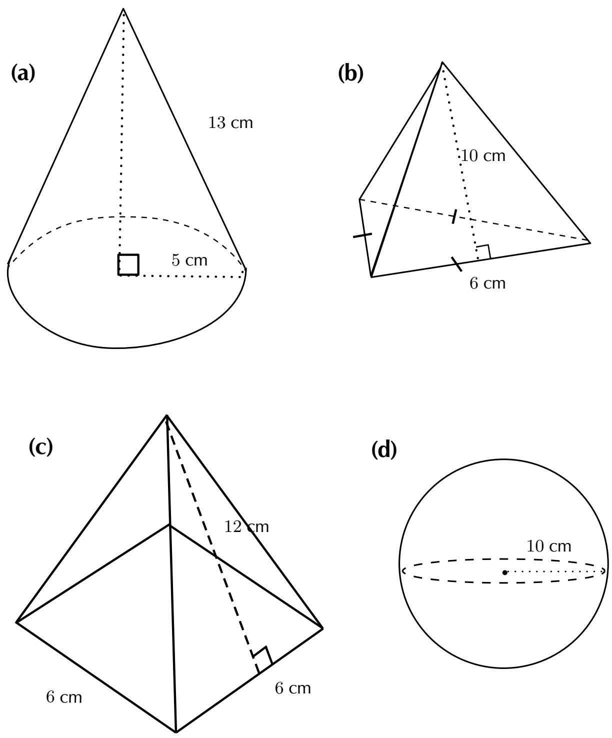
1. $V = L \times b \times h$
   $= 6 \times 7 \times 10$
   $= 420$ cm^3
2. $V = \frac{1}{2} \times h \times b \times H$
   $= \frac{1}{2} \times 5 \times 10 \times 20$
   $= 500$ cm^3
3. $V = \pi r^2 h$
   $= \pi(5)^2(10)$
   $= 785{,}4$ cm^3

Exercise 12 - 4

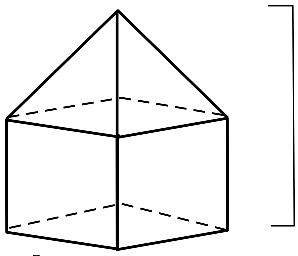
1. Find the total surface area of the following objects (correct to 1 decimal place if necessary):



2. Find the volume of the following objects (round off to 1 decimal place if needed):



3. The solid below is made up of a cube and a square pyramid. Find it's volume and surface area (correct to 1 decimal place):



Solutions to Exercise 12 - 4

1. (a) Surface area = area of base + area of walls
   $= \pi r(r + h_s)$
   $= \pi 5(5 + 13)$
   $= 282{,}74$ cm^2

   (b) We first need to find h_b:
   $(h_b)^2 = (b)^2 - (\frac{b}{2})^2$
   $= 36 - 9$
   $= 27$
   $h_b = 5{,}20$ cm
   Surface area = area of base + area of triangular sides
   $= \frac{1}{2}b(h_b + 3h_s)$
   $= \frac{1}{2}6(5{,}20 + 10)$
   $= 45{,}6$ cm^2

   (c) Surface area = area of base + area of triangular sides
   $= b(b + 2h)$
   $= 6(6 + 2(12))$
   $= 180$ cm^2

   (d) Surface area $= 4\pi r^2$
   $= 4\pi(10)^2$
   $= 1\ 256{,}64$ cm^2

2. (a) Volume $= \frac{1}{3} \times \pi(r)^2 \times H$
   $= \frac{1}{3}\pi(5)^2(13)$
   $= 108{,}33$ cm^3

   (b) We first need to find h:
   $(h)^2 = (b)^2 - (\frac{b}{2})^2$
   $= 36 - 9$
   $= 27$
   $h = 5{,}20$ cm
   Volume $= \frac{1}{3} \times \frac{1}{2}bh \times H$
   $= \frac{1}{3} \times \frac{1}{2}(27)(6) \times (10)$
   $= 270$ cm^3

(c) Volume $= \frac{1}{3} \times b^2 \times H$
$= \frac{1}{3}(6)^2(12)$
$= 144$ cm^3

(d) Volume $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}\pi(10)^3$
$= 4\ 188{,}79$ cm^3

3. Volume $=$ volume of cube $+$ volume of square pyramid
$= (5 \times 5 \times 5) + \frac{1}{3}(5 \times 5 \times (11 - 5))$
$= 175$ cm^3
Surface area $= 5$(sides of cube) $+ 4$(triangle faces of pyramid)
$= 5(5 \times 5) + 4(\frac{1}{2} \times 5 \times \frac{13}{2})$
$= 190$ cm^2

Exercise 12 - 5

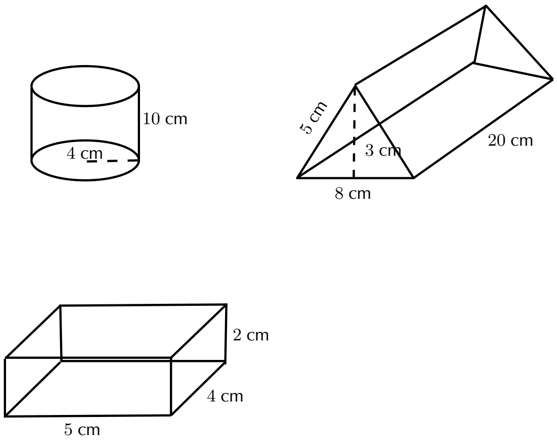
1. If the height of a prism is doubled, how much will its volume increase?
2. Describe the change in the volume of a rectangular prism if:
   (a) length and breadth increase by a constant factor of 3
   (b) length, breadth and height are multiplied by a constant factor of 2
3. Given a prism with a volume of 493 cm^3 and a surface area of $6\ 007$ cm^2, find the new surface area and volume for a prism if all dimensions are increased by a constant factor of 4.

Solutions to Exercise 12 - 5

1. Volume of prism = area of base $\times$ height of prism
new volume = area of base $\times 2 \times$ height
Therefore, the new volume is twice as much as the old volume if the height is doubled.
2. (a) Volume will be 9 times bigger ($3 \times 3 = 3^2 = 9$).
   (b) Volume will be 27 times bigger ($3 \times 3 \times 3 = 3^3 = 27$).
3. Volume $= 493 \times 4^3$
$= 31\ 552$ cm^3
Surface area $= 6\ 007 \times 4^2$
$= 96\ 112$ cm^2

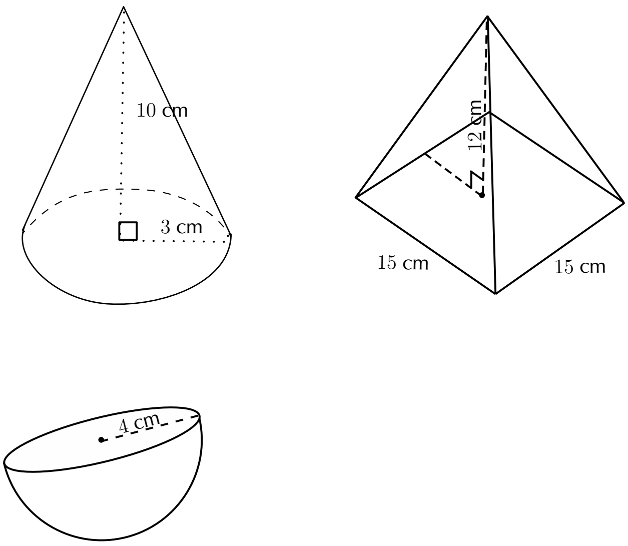
End of Chapter Exercises

1. Consider the solids below and answer the questions that follow (correct to one decimal place, if necessary):



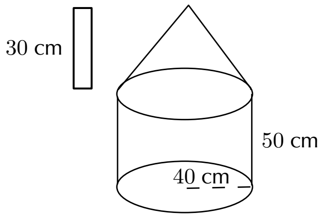
   (a) Calculate the surface area of each solid.
   (b) Calculate volume of each solid.
   (c) If each dimension of the solid is increased by a factor of 3, calculate the new surface area of each solid.
   (d) If each dimension of the solid is increased by a factor of 3, calculate the new volume of each solid.

2. Consider the solids below:



   (a) Calculate the surface area of each solid.
   (b) Calculate the volume of each solid.

3. Calculate the volume and surface area of the solid below (correct to 1 decimal place):



Solutions to End of Chapter Exercises

1. **(a)** Cylinder:
   Surface area $= 2\pi r^2 + 2\pi rh$
   $= 2\pi(4)^2 + 2\pi(4)(10)$
   $= 351{,}9$ cm^2
   Triangular prism:
   Surface area $= 2(\frac{1}{2}b \times h) + 2(H \times S) + (H \times b)$
   $= 2(\frac{1}{2}(8)(3)) + 2(20 \times 5) + (20 \times 8)$
   $= 384$ cm^2
   Rectangular prism:
   Surface area $= 2[(L \times b) + (b \times h) + (L \times h)]$
   $= 2[(5 \times 4) + (4 \times 2) + (5 \times 2)]$
   $= 72$ cm^2

   (b) Cylinder:
   Volume $= \pi r^2 h$
   $\pi(4)^2(10)$
   $= 502{,}7$ cm^3
   Triangular prism:
   Volume $= \frac{1}{2} \times h \times b \times H$
   $= \frac{1}{2}(3)(8)(20)$
   $= 240$ cm^3
   Rectangular prism:
   Volume $= L \times b \times h$
   $= 5 \times 4 \times 2$
   $= 40$ cm^3

   (c) Cylinder:
   Surface area $= 2\pi(3r)^2 + 2\pi(3r)(3h)$
   $= 2\pi 9(4)^2 + 2\pi(9)(4)(10)$
   $= 3\ 166{,}7$ cm^2
   Triangular prism:
   Surface area $= 2(\frac{1}{2}b \times h) + 2(H \times S) + (H \times b)$
   $= 2(\frac{9}{2}(8)(3)) + 18(20 \times 5) + 9(20 \times 8)$
   $= 3\ 456$ cm^2
   Rectangular prism:
   Surface area $= 2[9(L \times b) + 9(b \times h) + 9(L \times h)]$
   $= 2[9(5 \times 4) + 9(4 \times 2) + 9(5 \times 2)]$

$= 684$ cm^2

(d) Cylinder:
Volume $= \pi(3r)^2 3h$
$\pi(3(4))^2(3(10))$
$= 13\ 571{,}9$ cm^3
Triangular prism:
Volume $= \frac{1}{2} \times h \times b \times H$
$= \frac{27}{2}(3)(8)(20)$
$= 6480$ cm^3
Rectangular prism:
Volume $= 27(L \times b \times h)$
$= 27(5 \times 4 \times 2)$
$= 1080$ cm^3

2. **(a)** Cone:
Surface area $= \pi r^2 + 2\pi r\sqrt{r^2 + h^2}$
$= \pi(3)^2 + 2\pi(3)\sqrt{(3)^2 + (10)^2}$
$= 225$ cm^2
Square pyramid:
Surface area $= 4(\frac{1}{2}bh) + A$
$= 4(\frac{1}{2}(15)(12)) + 15^2$
$= 585$ cm^2
Half sphere:
Surface area $= \dfrac{4\pi r^2}{2}$
$= 100$ cm^2

(b) Cone:
$V = \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}\pi(3)^2 \times 10$
$= 94$ cm^3
Square pyramid:
$V = \frac{1}{3}A \times h$
$= \frac{1}{3}(15)^2 \times 12$
$= 900$ cm^3
Half sphere:
$V = \dfrac{(\frac{4}{3}\pi r^3)}{2}$
$= \dfrac{(\frac{4}{3}\pi(4)^3)}{2}$
$= 134$ cm^3

3. Surface area:
Cylinder:
Surface area $= 2\pi r^2 + 2\pi rh$
$= 2\pi(40)^2 + 2\pi(40)(50)$
$= 22\ 619{,}5$ cm^2
Cone:
Surface area $= 2\pi r\sqrt{r^2 + h^2}$
$= 2\pi(40)\sqrt{40^2 + 30^2}$
$= 12\ 566{,}4$ cm^2
Total surface area $= 22\ 619{,}5 + 12\ 566{,}4 = 35\ 185{,}9$ cm^2

Volume:
Cylinder:
$V = \pi r^2 h$
$= \pi(40)^2(50)$
$= 251\ 327{,}4$ cm^3
Cone:
$V = \frac{1}{3}\pi r^2 h^2$
$= \frac{1}{3}\pi(40)^2(50)^2$
$= 83\ 775{,}8$ cm^3
Total volume $= 251\ 327{,}4 + 83\ 775{,}8 = 335\ 103{,}2$ cm^3