FOUNDATIONS OF ALGEBRA

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THE COMPLETE TEACHER

Foundations of Algebra

What is lit?

Algebraic thinking involves finding and describing patterns, making generalisations about numbers, using symbols and models to represent patterns, quantitative relationships, and changes over time. Here are some typical algebraic expressions:
Distance = Rate x Time
total Apples = num Trees x apples Per Tree
a2 + b2 = c2

Why is it important?

The National Council for Teachers of Mathematics (NCTM) has extended the algebra standards to pre-school, citing research that algebraic concepts need to be grounded in extensive experience and developed over a long time (Sfard, 1991).

Teaching these concepts early using the language in which most of mathematics is communicated provides students with a solid foundation for understanding more ambitious mathematical thinking in the higher grades.

Younger student should start learning algebraic concepts such as patterns, multiple representations, and modeling mathematical relationships, such as change over time. Expanding the amount of time students have to explore algebraic concepts and abstract ways of thinking increases their chances of success. NCTM considers algebraic representation a prerequisite to formal work in "virtually all mathematical subjects including statistics, linear algebra, discrete mathematics, and calculus" (National Council of Teachers of Mathematics, 1989).

How can you make it happen?

The important algebraic concepts for elementary students to understand are variables, patterns and relationships, equality, constants, and change.

Variables

Variables can represent a range of values-numbers that vary-or for an unknown value. Variables typically are represented by italic symbols or letters, such as x, y, or more usefully in words, such as numApples. For example, consider the following problem:
If each tree in an apple orchard produces an average of 325 apples each season, then what is the total number of apples an orchard produces each season?

Start by representing this problem in words.
The total number of apples depends on the number of trees, which can be different for different orchards. If we want to find out how many apples there are, we have to know the number of trees in the orchard. The number of apples is equal to the number of trees times 326.

Once students have an understanding of how to state the mathematical relationships in words, then have them represent their thinking by using variables. The following are simple mathematical models of the relationship between apples and trees in an orchard.
number of apples = number of trees x 325
Or
numApples = numTrees x 325
Variables can be used to represent specific values. For example, if we know the number of trees is 100, we can substitute that information in the expression.
numApples = 100 x 325
numApples = 32,500

When variables represent specific values, they are subject to the same principles and rules of mathematics as numbers. For this reason, variables can be used in mathematical expressions to describe all manner of patterns, relations, or functions. For example, variables can be divided.
numApples⁄numTrees = 325

Encourage students to be clear about what each variable represents (the weight, length, cost, etc.) and to use variable names that convey the meaning of the values they represent. For example, in making mathematical models understandable, this expression:

Total cost = price per person X number of tickets
might be easier to understand than this expression:
c = p x t

Some single-letter variables names are conventional such as x and y for horizontal and vertical components of a point on a graph, r for radius, C for circumference, and a, b, and c for base, height, and hypotenuse of a right triangle.

Function machines

Using the concept of a "function machine" is an excellent way to demonstrate algebraic concepts to students. A function machine is something that takes in a value, does something to it, and generates a new value based on a rule or operation. A function machine shows students how different values for variables change the results of a situation, and can be any series of operations that demonstrate a pattern. For example, suppose you are having friends over for dinner, and estimate that each friend will eat two hot dogs. You can figure out how many hot dogs you need by constructing a function machine.

Patterns and relationships in tables

A data table is another common way of studying algebraic relationships. For example, imagine a "mystery" function machine in which students input a number of friends and the function machine outputs the number of hot dogs.

Number of Friends	**Number of Dogs**
2	4

3	6
4	8
5	10

What is the relationship between the numbers in the table? What pattern do you see? How many hotdogs will be needed if 155 friends attend the party?
Careful study of the table shows that the number of hot dogs is equal to the number of friends multiplied by two. This relationship can be represented by an expression such as:
numHotdogs = numFriends x 2
If 115 friends are expected at the party, then
numHotdogs =115 x 2
numHotdogs= 232

Equality

Another important concept for students to understand is algebraic equality. Young students may be accustomed to having the numbers and characters on the left side of the equal sign represent an arithmetic operation, the equal sign mean, "Here comes the answer," and the numbers and characters on the right side of the equal sign represent the answer. Students should be taught to view the equals sign as a symbol of equivalence and balance. The equal sign simply expresses that both mathematical phrases on either side of it are equal, not necessarily that either phrase is an answer.
In other words,
numHotdogs = numFriends x 2
is mathematically equivalent to
numFriends x 2 = numHotdogs

To further illustrate the idea of algebraic equality, consider a balanced scale with an apple on the left side of the scale and two bananas on the right side of the scale. In this example, we assume the weight of the apple is equal to the weight of two bananas. To verbalize this to students, say, "The weight of one apple is equal to the weight of two bananas."

The following expression can be used to represent this equality algebraically:
apple weight = banana weight + banana weight
Assuming the two bananas are of equal weight, then
apple weight = 2 x banana weight
To find the value of the weight of a banana, we can divide. When an operation is performed on one side of an equal sign, the same operation must be performed on the other side of the equal sign to keep the equation balanced. The results of dividing both sides of the equation by two are shown below.
apple weight = 2 × banana weight
apple weight / 2 = 2 × banana weight / 2
apple weight / 2 = 1 × banana weight
apple weight / 2 = banana weight

Constants

A constant is a value that does not change. For example, in the expression
Fahrenheit = Celsius x 1.8 + 32
Fahrenheit and Celsius are variables, while 1.8 and 32 are constants.

In the hot dog example, one constant is the number of hot dogs you will buy for each invited friend, or two hot dogs. If your brother eats three hot dogs, then you will always need one extra hot dog, regardless of the number of friends you invite. The number of hot dogs for your brother is a constant, because that number will not change. Based on this, you can construct a new data table that shows the total number of hot dogs you need.

Number of Friends	Number of Hot Dogs for Friends	Number of Hot Dogs for Brother	Total number of Hot Dogs

2	4	1	5
3	6	1	7
4	8	1	9

The first column describes the number of friends invited-a variable. The second column is the number of friends multiplied by two (the number of hot dogs per friend- a constant), the third column is the number of hot dogs for your brother, one (a constant), and the fourth column is the number of hot dogs for your friends plus one (the number needed for your brother).

This can be stated as, "The number of hot dogs needed is equal to two times the number of friends plus one."
This is the algebraic equation that explains the rule:
totalHotdogs = (2 x numFriends) + 1

Change

Algebra is also used to study change in different situations. Algebraic equations can be derived to describe how quickly or slowly something changes such as the change in a student's weight or height over the years. For young students, having the ability to interpret graphs that describe change, either at a variable rate or constant rate, is an important skill that will prepare them to both derive and interpret change situations in higher-level algebra. By having students create and analyze change represented by tables or graphs, they can determine patterns for the rate of change. The following data table:

Height of Jack's Beanstalk

Day	Height (feet)
1	2

2	5
3	10
4	17
5	26
6	37

can also be represented as a line graph or as an equation:

stalkHeight = day x day + 1

stalkHeight = day2 + 1

or as a function machine.

How can you stretch students' thinking?

To challenge students' thinking, model word problems that have multiple answers, and discuss algebraic expressions one could use to solve them. Have students use formulas in a spreadsheet to build simple mathematical models of word problems.

Another way to stretch student's thinking is to make data tables that represent the results of functions, and then have students create graphs from the data. Students can then interpret the graphs, asking questions and analyzing the changes and constants that are represented graphically.

When can you use it?

Reading/English

Have students compare the readability of texts and plot the reading difficulty on a chart, creating a scatterplot. Students will find variables, such as the average number of sentences and syllables per word, creating functions to find the readability of texts.

Writing

Once students have constructed algebraic models, have them provide written descriptions of how they developed the model and how it works, illustrated with appropriate graphs, tables, and mathematical expressions.

Math

One of the best ways to engage students is to have them collect data, identify patterns, create mathematical models, and then test those models by considering the data they generate.

Social studies

Have students collect statistical data (e.g., population growth, crime statistics, agricultural statistics, or sports statistics), display the data in multiple forms (e.g., graphs and tables), and then create mathematical models that approximate the relationships across values. For example, have students look at total population growth in their state over time compared to the population growth of the United States as a whole. Then ask them if the relationship be expressed by a formula.

Science

Students can graph information from an experiment. For example, have students graph the air temperature in their state on the first day of each month of the year. Have them interpret the graph and discuss why the temperature changes throughout the year. Ask them to hypothesize about why the temperature changes more rapidly during certain months than others. Do the same for different regions around the world and compare the results. Can students develop a formula or function machine that "guesses" the temperature, if given the month of the year?

Have students use scales to experiment with balance and equality. Have them explore the weight of various objects and create equations based on their results. For example, "The weight of one stapler is equal to the weight of two cups, one book, and eight pencils." Then have students use variables to record their equations and describe their equations in writing.

Source:

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