Problem Solving: Find a Pattern

What is it?

Finding a Pattern is a strategy in which students look for patterns in the data in order to solve the problem. Students look for items or numbers that are repeated or a series of events that repeat. The following problem can be solved by finding a pattern:
There are 1000 lockers in a high school with 1000 students. The first student opens all 1000 lockers; next, the second student closes lockers 2, 4, 6, 8, 10, and so on up to locker 1000; the third student changes the state (opens lockers that are closed, closes lockers that are open) of lockers 3, 6, 9, 12, 15, and so on; the fourth student changes the state of lockers 4, 8, 12, 16, and so on. This continues until every student has had a turn. How many lockers will be open at the end?

Why is it important?

Patterns are often introduced to students without the context of a word problem as in the following example: "Find a pattern in this sequence, explain how it works, and use that pattern to predict the next four numbers. 7, 10, 13, 16, 19, __, __, __, __."
Younger students often discover and continue using patterns that employ geometric shapes. For example, yellow circle, red square, green triangle, yellow circle, red square, green triangle, and so on.

Discovering patterns can help students learn multiplication facts when they notice that 4 x 7 is the same as 7 x 4 and that all numbers in the 10s column end with a zero.
The Find a Pattern strategy can be used to solve many math problems and can be used in combination with many other strategies, including making a table, making a list, or simplifying the problem.

How can you make it happen?

Introduce a problem to students that require them to find the pattern in order to solve the problem. For example:
If you build a four-sided pyramid using basketballs and don't count the bottom as an aside, how many balls will there be in a pyramid that has six layers?

Using cooperative learning groups to find solutions to problems helps students verbalize their thinking, brainstorm ideas, discuss options, and justify their positions. After finding a solution, each group can present it to the class, explaining how they reached their solution and why they think it is correct.

Or, students can explain their solutions in writing, and the teacher can display the solutions. Then students can circulate around the room to read each group's solution.

1. Understand the Problem
   Demonstrate that the first step to solving a problem is understanding it. This involves identifying the key pieces of information needed to find the answer. This may require students to read the problem several times or put the problem into their own words.
   Sometimes you can solve a problem just by recognizing a pattern, but more often you must extend the pattern to find the solution. Making a number table can help you see patterns more clearly.
   In this problem, students understand:

The top layer will have one basketball. I need to find how many balls there will be in each layer of a pyramid, from the first to the sixth. I need to find how many basketballs will be in the entire pyramid.

2. Choose a Strategy
   To use this strategy successfully, you need to be sure the pattern will really continue. Have students give reasons why they think the pattern is predictable and not based on probability. Problems that are solved most easily by finding a pattern include those that ask students to extend a sequence of numbers or to make a prediction based on data. In this problem, students may also choose to make a table or draw a picture to organize and represent their thinking.

Finding a Pattern is an appropriate strategy to use to solve the problem. This is a pattern that is predictable and will continue.

3. Solve the problem
   Start with the top layer or one basketball. Determine how many balls must be under that ball to make the next layer of a pyramid. Use manipulatives if needed. Students can use manipulatives of any kind, from coins to cubes to golf balls.

   Students can also draw pictures to help them solve the problem.
   You may want to have groups use different manipulatives and then compare their solutions to determine whether the type of manipulative affected the solution. If students are younger, start with three layers and discuss their answers to this simpler problem. Then move on to more layers as students gain an understanding of how to solve the problem.

Layer	Balls Added	Balls in This Layer

1 (top)	1	1
2	3	4 (1 + 3 = 4)
3	5	9 (4 + 5 = 9)
4	7	16 (9 + 7 = 16)
5	9	25 (16 + 9 = 25)
6	11	36 (25 + 11 = 36)

4. If it helps to visualize the pyramid, use manipulatives to create the third layer. Record the number and look for a pattern. The second layer adds 3 basketballs and the next adds 5 basketballs. Each time you add a new layer, the number of basketballs needed to create that layer increases by 2.
    1. 1
    2. 1 + 3 = 4
    3. 4 + 5 = 9
    4. Continue until six layers are recorded. Once a pattern is found, students might not need to use manipulatives.9 + 7 = 16
    5. 16 + 9 = 25
    6. 25 + 11 = 36

5. Then add the basketballs used to make all six layers. The answer is 91 balls.Look at the list to see if there is another pattern. The number of balls used in each level is the square of the layer number. So the 10th layer would have 10 x 10 = 100 balls.

6. Check
   Read the problem again to be sure the question was answered.
   Yes, I found the total number of basketballs in the six-layer pyramid.
   Check the math to be sure it is correct.
   1 + 4 + 9 + 16 + 25 + 36 = 91

   Determine if the best strategy was chosen for this problem, or if there was another way to solve the problem.
   Finding a pattern was a good way to solve this problem because the pattern was predictable.

7. Explain
   Students should explain their answer and the process they went through to find it. It is important for students to talk or write about their thinking. Demonstrate

how to write a paragraph describing the steps students took and how they made decisions throughout the process.

8. First, I started with the first layer. I used blocks to make the pyramid and made a list of the number of blocks that I used. Then I created a table to record the number of balls in each layer.

Layer	Balls Added	Balls in This Layer
1 (top)	1	1
2	3	4 (1 + 3 = 4)
3	5	9 (4 + 5 = 9)
4	7	16 (9 + 7 = 16)
5	9	25 (16 + 9 = 25)
6	11	36 (25 + 11 = 36)

9. I made four layers and then saw a pattern. I saw that for each layer, the number of balls used was the number of the layer multiplied by itself. I finished the pattern without the blocks, by multiplying the number of balls that would be in layers 5 and 6.
   Then I added up all of the balls in each layer.
   1 + 4 + 9 + 16 + 25 + 36 = 91

   I got a total of 91 basketballs.

10. Guided practice
    Have students solve the following problem using the strategy of Find a Pattern.
    A woman is trying to cut down the number of cans of soda she drinks each week. She makes a plan so that in several weeks she will be drinking only one can of soda. If she starts with 25 cans the first week, 21 cans the second week, 17 cans the third week, 13 cans the fourth week, and continues this pattern, how many weeks will it take her to reach her goal?
    Have students work in pairs, in groups, or individually to solve this problem. They should be able to tell or write about how they found the answer and justify their reasoning.

How can you stretch this strategy?

Math problems can be simple, with few criteria needed to solve them, or they can be multidimensional, requiring charts or tables to organize students' thinking and to record

patterns. In using patterns, it is important for students to find out if the pattern will continue predictably. Have students determine if there is a reason for the pattern to continue, and be sure students use logic when finding patterns to solve problems.

- For example, if it rains on Sunday, snows on Monday, rains on Tuesday, and snows on Wednesday will it rain on Thursday?
- Another example: If Lauren won the first and third game of chess, and Walter won the second and fourth game, who will win the fifth game?
- Another example: If a plant grew 13 centimetres in the first week and 10 centimetres in the second week, how many centimetres will it grow in the third week?

Because these are questions of probability or nature, be sure students understand why patterns can't be used to find these answers.

Source: